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PREFACE

The public consciousness of the importance of the nuclear power as a viable energy option and its awareness of the potential safety issues have made nuclear power an area of enormous interest to the technical community. Thermal-hydraulics, i.e., the transport of energy and flow, is a major engineering discipline that deals with the performance of the nuclear power systems and the operation of nuclear plant components. This bound volume is a compilation of the papers presented at the ASME sessions on "Thermal-Hydraulics in Nuclear Power Technology" at the 20th National Heat Transfer Conference. The sessions are organized by the ASME K-13 Nucleonics Heat Transfer Committee of the Heat Transfer Division. This volume represents an advance of the state-of-art of the heat transfer field.

This volume covers the fundamental theories and phenomena, the performance characteristics of reactor components as well as the behavior of simulated nuclear systems and test reactors. In the fundamental area, the papers are associated with the heat transfer due to condensation in counter-current flooding, the hydrodynamics of a submerged vapor jet, the hydrodynamic criterion of flooding, the prediction of boiling transition, and the stripping at the interface of thermal stratification. The papers that deal with the performance characteristic of reactor components include the study of the transient analysis of accumulators and the modes of natural circulation in U-tube steam generators for light water reactors and thermal fluctuations in the steam generator of a fast breeder reactor. The papers in the category of plant system behavior are the study of the response of a scaled boiling water reactor system during a loss-of-coolant accident and the modeling of turbulent flow in the Fast Flux Test Facility.

It is hoped that this bound volume does not merely represent another addition to the rapid-growing literature, but to advance the understanding and the capability of modeling the nuclear power systems.

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CONDENSATION HEAT TRANSFER AND FLOODING IN A COUNTER-CURRENT SUBCOOLED LIQUID AND SATURATED VAPOR FLOW

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ABSTRACT

The paper develops an analysis of flooding with condensation heat transfer in a vertical tube. The two-phase flow pattern is assumed to be annular with the subcooled liquid adjacent to the tube surface. The core of the tube consists of a homogeneous mixture of saturated vapor and liquid droplets.

Examination of the void fraction envelope of the combined heat transfer and hydrodynamic equations revealed the existence of a flooding solution in which the parameters of its description are the ratio of gravity to viscous forces, buoyancy number, Bond number, liquid droplets entrainment parameter, subcooling number, Prandtl number, and the tube length parameter. Heat transfer results revealed that higher volumetric flooding fluxes are possible compared to the adiabatic flow. At higher subcooling numbers the results also showed a hysteresis effect which is more pronounced at lower Prandtl numbers and for shorter tubes.

The adiabatic flooding solution compared well with the experimental data for air-water and air-oil for smooth tube ends. For non-smooth tube ends, good comparison is achieved at higher liquid volumetric fluxes. The heat transfer flooding results compared satisfactorily with the experimental data of steam and water.

NOMENCLATURE

A  Flow cross-sectional area
Bo  Buoyancy number, 1−Dp/cp
Cp  Specific heat at constant pressure
D  Tube internal diameter
D*  Tube diameter parameter, D/\sqrt{\frac{g(\rho_l−\rho_g)}{\sigma}}
, or Bond number
e  Entrainment parameter, [M_l/M_v]
F  Friction coefficient
g  Gravitational constant
h  Local heat transfer coefficient
H  Average heat transfer coefficient, \int_h dz/L
h_e  Enthalpy of evaporation
H_t  Constant in equation (15)
H_w  Constant in equation (13)
J  Volumetric flux, M/(\rho A)
J*  Non-dimensional volumetric flux, defined by equation (17)
k  Thermal conductivity
L  Tube length
M  Mass flow rate
N_l  Ratio of gravity to viscous forces parameter, \frac{(gD^2)(\rho_l−\rho_g)}{\sigma}
Nu  Nusselt number, h_d/k_l
p  Static pressure
Pr  Prandtl number
q  Heat flux
Re  Film Reynolds number, \frac{Dp}{\nu_l}
Sc  Subcooling number, \frac{C_p(T_s−T_{liq})}{h_l}
St  Stanton number, \frac{Nu}{Re_{liq}Pr}
T  Liquid film bulk temperature
T_o  Reference temperature
W  Average fluid velocity
x  Axial distance along the tube, Figure 1

Greek Symbols

\alpha  Void fraction, defined by equation (11)
\eta  Condensation efficiency, defined by equation (7)
\mu  Absolute viscosity
\xi  Perimeter
\rho  Density
\sigma  Surface tension
\tau  Shear stress

Subscripts

c  Pertains to the core
d  Pertains to the liquid droplets
g  Gas or vapor	
f  Liquid film-vapor core interface
i  Pertains to the liquid film
li  Pertains to the liquid film at the entrance to the tube
lt  Liquid film and liquid droplets
s  Saturation value

I. INTRODUCTION

Condensation of saturated or superheated vapor onto a counter-current flowing subcooled liquid has an application in the design of a pressurized water reactor (PWR) downcomer. When the loss-of-coolant accident (LOCA) occurs in the reactor, due to the break of a pipe in the primary coolant system, emergency core cooling (ECC) water is injected into the annular space which surrounds the reactor core in order to flood and cool the core. Some of the steam which is generated in the reactor core attempts to exit from the core through the annular space in a counter-current flow arrangement. The upflowing steam condenses onto the downflowing injected water, and if the steam flow-rate is sufficiently large, the steam flow limits the water supply rate to the reactor core. Under such a flooding situation, core cooling can be greatly hampered, and with further increase in the steam flow, the injected water is bypassed out from the broken leg. The design of ECC must prevent the occurrence of this situation with the proper design of an ECC system.

The understanding of fluid mechanics and heat transfer in a flooding situation is, therefore, of considerable interest, and their relative importance has not been fully investigated.
Flooding in the presence of heat transfer has been studied by Liu, Collier, and Cumings (1), Segov and Collier (2), and Liu and Collier (3) for the purpose of understanding the FWR downcomer thermal-hydraulics. These authors choose an annular flow configuration with liquid which flows adjacent to the downcomer walls and with saturated vapor which flows upwards through the annular core region. In Ref. 1, a heat transfer analysis is developed for the liquid film bulk temperature distribution as a function of the local liquid film mass flow rate and the liquid film mass flow rate at the inlet into the downcomer annulus. An expression was also developed for the local mass flow rate of the condensed vapor in terms of the 'condensation efficiency' or the heat transfer coefficient. Flooding in the presence of heat transfer was accounted for by modifying the Wallis's (4) adiabatic flooding correlation for the local change in the vapor volumetric flux, i.e.

\[
J_g^* - \eta L_J^* = \frac{\rho_f}{\rho_l} J_g^* + \frac{\rho_f}{\rho_l} = \text{constant}.
\]  

(1)

In the experiments of Liu et al. (1) no significant difference in flooding fluxes is found between the steam-water at low subcoolings and the air-water mixture in an adiabatic flow. In a later study, Segov and Collier (2) determined that the empirical coefficients which enter into the flooding model of equation (1) are test-geometry dependent, and at large subcoolings that the comparison between the analysis and the experiment is not as satisfactory as at low liquid subcoolings.

Recently, Liu and Collier (2) carried out an experimental study of the counter-current condensation of steam onto the subcooled water in an adiabatic tube and determined a correlation for the average heat transfer coefficient. They also observed a hysteresis effect in the flooding curves when the steam flow was increased from zero value to the point of complete liquid by-pass and when it was decreased to zero again. Tien (5) has alluded to the possibility of such a hysteresis' effect in the heat transfer flooding curves by examining equation (1) for different values of the liquid subcooling.

Counter-current condensation with flooding provides very interesting physics, and much of this physics is at the present incompletely understood. In part this is because of the complexity in carrying out accurate physical modeling of flooding phenomena and because of the absence of the experimental data. In flooding without heat transfer in a tube, flooding is arrested by the tube inlet configurations (Wallis (4), Tien et al. (6) and Bharathryan et al. (7)), and on the methods by which the two-phases are introduced into the tube. With the addition of heat transfer, the local momentum balance in the flow field is continuously readjusted to account for the interphase momentum transfer which results from the condensing mass flux. In the heat transfer experiments carried out to date, no significant difference between heat transfer and adiabatic flooding curves is observed, and no analysis has been developed from fundamental principles in order to investigate the parameters which effect flooding in the presence of heat transfer.

Problem Under Investigation and Objectives of the Paper

The paper has the objective to extend the study of heat transfer and flooding hydrodynamics in a tube with an adiabatic wall in which saturated vapor is condensing onto a counter-current flowing subcooled liquid. The flow model is illustrated in Figure 1. The model consists of a time invariant liquid film thickness which is adjacent to the tube wall, and of a homogeneous mixture of liquid droplets which are entrained in the saturated vapor core. The positive flow direction is vertically downward and at the inlet the following assumptions are made in the analysis:

1. The flow field can be described by two velocity fields, one for the liquid film and the other for the vapor core.
2. Viscous dissipation and kinetic energy effects are negligible.
3. Pressure is uniform across the tube cross-sectional area.
4. The vapor core enthalpy is primarily due to the saturated vapor.
5. There is no axial conduction in the liquid film.
6. The interface between the liquid film and the vapor core is, on the average, smooth.
7. Constitutive equations for the wall friction and liquid film-vapor core interface friction, which are described in Section II, adequately represent the phenomena under investigation.
8. Fluid properties are constant.
9. Liquid film inertia, and liquid film and vapor core velocities have flat profiles.
10. Axial shear stresses are negligible.

With the foregoing considerations, the flow field is modeled by the one-dimensional form of conservation equations for mass, momentum and energy which are described, as for example, by Dobra (2).

The specific objectives of the paper are the following:

1. To develop heat transfer expressions for arbitrary values of the liquid subcoolings.
2. To derive an adiabatic flooding model where the parameters of its description are the ratio of gravity to viscous forces, the Bond number, the buoyancy number, and the liquid droplet entrainment parameter.
3. To show how the entrainment parameter can be utilized to explain the trend in the experimental data.
4. To develop flooding curves with condensing heat transfer, and to show how the additional parameters in the form of subcooling number, Prandtl number and the tube length to the diameter ratio enter into their description.

II. ANALYSIS

Heat Transfer Analysis

Conservation of mass for the two-phase mixture requires that

\[
\frac{d}{dz} (M_l + M_g) = 0,
\]

(2)

where \( M_l \) is the liquid film mass flow-rate and \( M_g \) is the core mass flow-rate. The later is the sum of liquid droplets and vapor mass flow-rates, \( M_g = M_d + M_w \).

Since the core flow is assumed saturated, only one energy equation is necessary to consider. For the two-phase mixture this equation is of the form

\[
\frac{d}{dz} [M_c (C_{p_c} (T_c - T_0) + M_c h_c)] = q_w T_w = 0.
\]

(3)

In the above expression, \( T_w \) is the liquid film tempera-
Fig. 1 Two-phase flow configuration in a tube. \( T_0 \) is the reference temperature, and \( h_w \) is the core enthalpy by assumption it is equal to the saturated vapor value.

Integrating equation (3) from \( z=0 \) where \( M_1 = M_{11} \) and \( T_1 = T_{11} \) to \( z>0 \), and utilizing equation (2) to eliminate \( M_2 \), yields the liquid film bulk temperature distribution along the tube, i.e. \[
\frac{T_1 - T_{11}}{T_S - T_{11}} = \left(1 - \frac{M_{11}}{M_1}\right) \frac{1 + S_{11}}{S_{11}}, \tag{4}
\]
where \( T_S \) is the saturation temperature and \( S_{11} \) is the liquid subcooling number defined by \[
S_{11} = \frac{C_p(T_S - T_{11})}{h_{lg}}. \tag{5}
\]

The local heat transfer coefficient, \( h \), can be defined from the liquid film-vapor core interface energy balance

\[
\ln\left(\frac{T_S - T_1}{T_S - T_{11}}\right) = \frac{dM_1}{dc} \left(\frac{h}{dS_{11}}\right), \tag{6}
\]
where \( l \) is the heat transfer perimeter and in this paper it is chosen as the ratio of tube flow area to the tube diameter, \( \pi D/4 \). The average heat transfer coefficient, \( h_{avg} = (\int_{T_1}^{T_S} dq/dz)/L \), and the average Nusselt number, \( N_u = h_{avg} D/\lambda_1 \), are obtained by integrating equation (5) and utilizing equation (4).

\[
St = \frac{\frac{Nu}{Re_{11} Pr_p}}{Re_{11} Pr_p} = \left[\frac{1 + S_{11}}{1 + S_{11}} \right] \left[\frac{T_1 - T_{11}}{T_S - T_{11}}\right] = \frac{S_{11}}{1 + S_{11}} \tag{7}
\]

\( Re_p \) is the liquid-film Reynolds number which is defined by

\[
Re_p = \frac{\frac{M_1 D}{\lambda_1}}{\frac{\rho_1}{u_1}}, \tag{8}
\]
and \( St \) is a form of the Stanton number.

Fig. 2 Relationship between the average Nusselt number and the liquid film bulk temperature for varying subcooling numbers.

Fig. 3 Relationship between the average Nusselt number and the film mass flow-rate for varying subcooling numbers.

A plot of equation (6) is illustrated in Figure 2 with the subcooling number as the parameter. At large temperature ratios, \( (T_1 - T_{11})/(T_S - T_{11}) \), which occur far downstream from the top of the tube, there are considerable differences between average Nusselt numbers at higher and lower subcooling numbers. The low subcooling number formulation of Ref. 1, therefore, underpredicts the heat transfer rate if it is applied to subcooling numbers of greater than about .1 in long tubes.

An expression which is useful for determining the average Nusselt number from the experimental measurement of the local film mass flow-rate is obtained by elimin-
nating the temperatures between equations (4) and (6). The result is

\[ S_{1f} = 1 - \frac{M_{11}}{M_{11}} + (1 + S_{11}) \ln \left[ \frac{S_{11}}{(1 + S_{11}) - \frac{M_{11}}{M_{11}}} \right] \tag{7} \]

and is plotted in Figure 3 for different values of the subcooling number \( S_{11} \). One additional useful expression is obtained by re-arranging equation (4) for the mass flow-rate as follows:

\[ \frac{M_{11}}{M_{11}} = \frac{1}{1 + \frac{T_{11} - T_{11}}{T_{1} - T_{11}}} + (1 + S_{11}) \tag{8a} \]

As expected, this equation gives the result that

\[ \lim_{T_{1} - T_{11} \to 0} \frac{M_{11}}{M_{11}} = 1 + S_{11} \quad \text{and thus} \]

\[ 1 < \frac{M_{11}}{M_{11}} < 1 + S_{11} \tag{8b} \]

For subcooling numbers which are much less than one, equations (4), (6) and (7) are reduced respectively to

\[ \frac{T_{1} - T_{11}}{T_{1} - T_{1}} = \frac{1}{1 + \frac{M_{11}}{M_{1}}} \tag{4'} \]

\[ \frac{\bar{N}_{u}}{Re_{11} Pr_{1}} = \ln(1 + \frac{T_{1} - T_{11}}{T_{1} - T_{11}}) \tag{6'} \]

\[ \frac{M_{1} - M_{11}}{M_{11}} = S_{11} \left[ 1 - \exp(-\frac{\bar{N}_{u}}{Re_{11} Pr_{1}}) \right] \equiv S_{11} n \tag{7'} \]

and are identical to the low subcooling number formulation in Ref. 1. Liu et al. (1) defined the term in square brackets in equation (7') as the condensation efficiency \( n \), since it represents the effectiveness of heat transfer process. Utilising the above definition of \( n \) in equation (7), it is possible to produce a plot which is illustrated in Figure 4. The dashed lines in this figure represent the plot of equation (7'), and for \( S_{1} < 0.5 \) the error incurred in using this equation instead of equation (7) is less than 10%. The equilibrium situation in Figure 4 is represented by \( n = 1 \), and it occurs when the liquid bulk temperature becomes equal to the saturation temperature. At this point \( M_{1} = (1 + S_{11}) M_{11} \). Also illustrated in Figure 4 are the experimental data of Liu et al. (1), and Lee et al. (9) from the co-current condensation study in a horizontal duct. These data indicate that in most cases full equilibrium flow is not achieved.

The heat transfer analysis presented above is equally valid for co-current and counter-current condensation of saturated vapor onto a subcooled liquid in adiabatic ducts of arbitrary cross-section. As such, the analysis can form the basis for data reduction for arbitrary values of the subcooling number and bring consistency to the large class of problems. The inconsistency in the literature with respect to the above remark is evident in References 3 and 9 where the average Nusselt number is computed from the experimentally measured values of the liquid mass flow-rates in completely different ways. The outcome of this inconsistency is reflected in heat transfer correlations which are presented in the literature.

**Fig. 4. The effect of condensation efficiency and subcooling number on the liquid film mass flow-rate**

The heat transfer analysis is complete when the variation of liquid mass flow-rate is expressed in terms of the axial distance along the tube. Liu and Collier (3) carried out extensive low-subcooling number measurements in counter-current condensing flows with steam and water in a tube and determined that the average Nusselt number can be represented by the following expression:

\[ \bar{N}_{u} = 0.00428 \frac{(2.1)_{11}^{14}}{D} \times 3.8 1_{11}^{14} \times 1.477 \tag{9} \]

The range of validity of equation (9) is as follows:

\[ S_{11} ; 0.07 - 0.14 \]

\[ N_{1} ; 4 \times 10^{-6} - 10^{-4} \]

\[ z/D ; 7 - 30 \]

\[ J_{1} ; 0.05 - 0.16 \tag{9b} \]

The non-dimensional group \( N_{1} \), (Wallis (4)) represents the ratio of gravity to viscous forces and appears more usefully in the hydrodynamic analysis of the next section. \( J_{1} \) is the non-dimensional liquid film volumetric flux and is defined by equation (17).

The combination of equations (7) and (9) yields an expression in which \( z/D \) is related to the liquid film mass flow-rate and to the independent parameters of the problem \( (N_{1}, S_{11}, Pr_{1}, Re_{11}) \). These equations will be used in the next section.

**Hydrodynamic Analysis**

Momentum equations can be written for the two-phase mixture, for the liquid film, and for the liquid droplets and vapor core. Since the formulation is carried out in terms of two velocity fields, only two of these equations need to be considered. By eliminating the pressure gradient between the momentum equation for the two-phase mixture and a momentum equation for the liquid film, the following equation is obtained:

\[ (a-1) \frac{\partial}{\partial z} \frac{1}{\rho_{L}} \frac{\partial}{\partial z} \frac{1}{\rho_{L}} \frac{dV}{dz} + (a-1) \frac{\partial}{\partial z} \frac{1}{\rho_{L}} \frac{\partial}{\partial z} \frac{1}{\rho_{L}} \frac{dV}{dz} = \frac{\partial}{\partial z} \left( \frac{\partial}{\partial z} \right) \left( \frac{1}{\rho_{L}} \frac{\partial}{\partial z} \right) \tag{10} \]
The first term on the left side of equation (10) represents the liquid-droplets and vapor inertia effect. The second term represents the momentum change of the vapor and droplets along the tube due to the condensation of vapor in the core, and $W_c$ in the same term is the velocity of the interface. Buoyancy and friction forces are included on the right side of equation (10). The void fraction $\alpha$ is defined by

$$\alpha = \frac{A_0}{A} = \frac{A_d + A_v}{A}, \quad (11a)$$

and the homogeneous core density is given by

$$\rho_c = \rho_\gamma(1 - \alpha_c) + \rho_g \alpha_c, \quad (12)$$

where $\alpha_c$ is the core void fraction. It is defined by the equation

$$\alpha_c = \frac{A_v}{A_c} \quad. \quad (11b)$$

The core inertia term in equation (10) can be expressed by

$$\frac{d(M_0 \rho_c)}{dz} = A_c \frac{d}{dz} \left( \frac{J_c^2}{\alpha} - \frac{A \rho_1 - \rho_g}{\alpha_c} \frac{d\alpha_c}{dz} + A_0 \frac{dp}{dz} \right),$$

where $J_c$ is the core volumetric flux and is related to $M_0$ by:

$$M_0 = \rho_o A J_c = \rho_\gamma(1 - \alpha_c) W_c,$$

$M_c$ is the two-phase Mach number and is given by

$$M_c^2 = \frac{\rho_o^2}{A_0^2} \frac{dp}{dz}.$$

Of the three terms on the right of the above equation, the first term is most significant. Its inclusion into the theory presented below is not simple and has been, therefore, neglected. In the case of large subcoolings this neglect might not turn out to be justifiable. The effect of variation of the core void fraction along the tube due to the liquid-droplets-entrainment and the effect of the compressibility of the vapor is expected to be of the second order in importance when compared to buoyancy and viscous forces, and were neglected in the analysis.

The expressions for interface velocity, wall shear stress and interfacial shear stress were chosen as follows:

1) Interface Velocity, $W_1$:

$$W_1 = I_1 W_1. \quad (13)$$

In the adiabatic co-current annular flow, plausible range of $I_1$ is 1 to 3, and is assumed to be valid in the present analysis (Wallis (10)).

2) The Wall Shear Stress, $\tau_w$. The wall shear stress is expressed by the usual relation

$$\tau_w = \left( W_1 \right)^{1/2} \left( \frac{J_c}{(1 - \alpha_c)^2} \right), \quad (14)$$

and neglecting the film thickness effect on the distribution of $f_w$ (Wallis (10)), the wall friction factor is represented by the equation

$$f_w = \frac{c}{\left( Re \right)^n}. \quad$$

For laminar flow in the liquid film $c=16$ and $n=1$, and for turbulent flow $c=0.079$ and $n=0.25$. The transition region is selected to be the intersection of laminar and turbulent representation of equation (14) which occurs at $Re=1,190$.

3) The Interfacial Shear Stress, $\tau$. In the co-current condensing flow inside vertical tubes, Linehan et al. (11) demonstrated that when the liquid film-vapor core interface shear stress $\tau$ is computed from adiabatic correlations, the best agreement between the analysis and the experiment is not achieved. Comparison of analysis with the experimental data indicates that the there should be a term included in $\tau$ which is proportional to the interphase momentum transfer, i.e.,

$$\tau = \frac{f_4}{\rho_c(W_c - W_1)} \frac{dM_1}{dz}$$

where $f_4$ is computed from adiabatic correlations. $I_1$ is a constant and equal to $-1$ for the counter-current flow of liquid in the film and condensing vapor in the core.

The form of equation (15) is plausible also in a Couette turbulent layer with suction (Kays and Crawford (12)) is considered in the region of vapor core in the vicinity of the liquid film-vapor core interface. The effect of condensation mass flux at the interface is to produce at the interface a 'suction velocity'. The suction velocity introduces a correction to the interfacial shear stress which would normally be present at the interface without the suction effect.

In the counter-current condensing flow, both terms on the right side of equation (15) are considered significant. Their relative importance is discussed in the next section.

The interface friction factor in flooding is larger than in either co-current liquid-gas upflow or downflow. Recent experimental data with air-water flooding (13) indicate that for wide range of liquid and gas volumetric fluxes and tube diameters, $f_4$ can be represented by the following relations:

$$f_4 = 0.005 + \frac{a D^2}{2} \left( 1 - \frac{\gamma}{\rho} \right)^b$$

$$\log 10^b = -56 + 9.07/D \quad (16)$$

$$b = 1.63 + 4.74/D. \quad$$

The dependence of $f_4$ on the Bond number $D^2$ is pronounced for smaller tube diameters, i.e. for $D<50$.

To complete the analytic description of the problem under investigation, it is useful to non-dimensionalize equation (10). Towards this end the core volumetric flux is defined by $J_c = \rho_\gamma W_0$, liquid film volumetric flux by $J_1 = (1 - \alpha_c) W_1$, and vapor volumetric flux by $J_v = \rho_\gamma W_1 = a J_1$. The volumetric fluxes are non-dimensionalized according to the expressions

$$\frac{J_0}{\rho_\gamma} = \frac{J_1}{\rho_\gamma (1 - \alpha_c)^2}, \quad \frac{J_v}{\rho_\gamma} = \frac{J_1}{\rho_\gamma (1 - \alpha_c)^2}. \quad (17)$$

and the non-dimensional axial coordinate in the tube is defined by $z = z_D$. Utilizing above definitions and equations (11) to (16), equation (10) becomes
In equation (21a), \( \frac{dJ^k}{d\alpha} \) is expressed in terms of \( \frac{ds^k}{d\alpha} \) if the continuity equation (2) is used. Hence

\[
\frac{dJ^k}{d\alpha} = -\rho_g \frac{\partial}{\partial \alpha} \left( \frac{\gamma^k}{\alpha} \right) \frac{ds^k}{d\alpha}.
\]  

(21c)

Equations (9) and (18) in (21a, b, c) are sufficient to investigate flooding in a tube in the presence of condensing heat transfer. The arbitrary parameters of the problem are the following:

- Ratio of gravity to viscous forces, \( N_L \).
- Density ratio (Buoyancy number), \( Bo=1-\rho_g/\rho_1 \).
- Bond number, \( D_b \).
- Subcooling number, \( S_{11} \).
- Prandtl number, \( Pr \).
- Entrainment parameter, \( e \).
- Dimensionless liquid volumetric flux at the tube inlet, \( J^k_{11} \).

Solution Procedure for Equation (18)

The flooding solution of equation (18) for the liquid and vapor volumetric fluxes was obtained by investigating the singularity set with respect to the void fraction \( \alpha \), or by setting

\[
\frac{dF}{d\alpha} = G = 0,
\]

(22)

and eliminating \( \alpha \) between these two equations. Examination of equations (18) and (22) revealed that a flooding solution exists for the downflowing liquid, \( J^k_{11} > 0 \), and for the upflowing vapor, \( J^k_{11} < 0 \). Since the equations are non-linear in \( J^k_{11} \) and \( J^k_{21} \), their simultaneous solution has been carried out numerically with the two-dimensional Newton-Raphson iteration scheme. This scheme is of the form

\[
\begin{pmatrix}
3F^k \frac{1}{J^k_{11}} \\
3G^k \frac{1}{J^k_{21}} \\
3F^k \frac{1}{J^k_{11}} \\
3G^k \frac{1}{J^k_{21}}
\end{pmatrix} \begin{pmatrix}
J^k_{11+1} - J^k_{11} \\
J^k_{21+1} - J^k_{21}
\end{pmatrix} + \begin{pmatrix}
\gamma^k \\
\gamma^k
\end{pmatrix} = 0,
\]

(23)

where \( k \) denotes the iteration number. In all results presented in the paper, the computed values of the liquid and vapor volumetric fluxes were considered correct when the error between two successive iterations was less than 10^{-6}. On the average, 15 iterations produced the desired error tolerance.

III. RESULTS AND DISCUSSION

Before accepting the heat transfer flooding results it is necessary to test the theory against the experimental data for flooding in a tube in adiabatic flow.

Adiabatic Flowing Solution

In adiabatic flow, \( J^k_{11}/J^k_{21}=0 \) in equation (18) and flooding is described by the following equations:

\[
-\rho_g \frac{J^k_{21}}{J^k_{11}} = 4a_1^k \left(1-a \right) \frac{df_{11}}{d\alpha} + \frac{1}{2} \frac{J^k_{11}}{J^k_{21}} \left( S_{11} + J^k_{21} \right) \frac{df_{11}}{d\alpha}.
\]

(24)

The average Nusselt number is expressed by equation (7), and in the now nonsaltsuro it is of the following form:

\[
\frac{J^k_{21}}{J^k_{11}} = \frac{J^k_{11} + (1+J^k_{21}) \ln \left( \frac{J^k_{21}}{J^k_{11}} \right)}{\nu_1 \beta_1 \alpha_1^k \left( 1 - \frac{6 \alpha_1 f_{11}}{4 \alpha_1 \left( 5 \alpha_1 - 2 a_1 \right) \frac{df_{11}}{d\alpha}} \right)} 
\]

(25)
Equations (24) and (25) assume that the variation of \( \phi \) with \( a \) is negligible as is indeed the case. Inspection of these equations indicates that the flooding solution exists for \( J_g > 0 \) and \( J_g < 0 \).

The adiabatic flooding solution for \( N_t = 12,000, D^* = 9.3 \) and \( Bo = 9988 \) is shown in Figure 5 in the plot of \( \sqrt{J_g} \) versus \( J_r \). The values of these parameters correspond to the air-water at atmospheric pressure and in .0254m diameter tube. The liquid volumetric flux represented in Figure 5 is the value composed of the liquid fluxes in the film and in the core, since it is this total value which is reported in experiments on flooding. The total liquid volumetric flux is related to the liquid film volumetric flux by \( J_{fl} = (1 + e) J_f \).

![Figure 5 Comparison of air-water experimental data with theory](image)

The ideal flooding line in Figure 5 corresponds to the zero value of the entrainment parameter, and is seen to be only in qualitative agreement with the experimental data of Wallis (4) and Bharathan et al. (7). For non-zero value of the entrainment parameter, the analysis captures the experimental data very well except for tubes with sharp water inlet ends at low liquid volumetric fluxes. The entrainment of liquid in the gas or vapor core is caused by the breakdown of waves on the liquid-gas interface and at the tube ends where these ends are poorly streamlined. For air-water flooding data with smooth tube ends, the analysis agrees with the experiment if the value for an entrainment parameter is chosen to be .1. This value is reasonable, and future experimental studies should attempt the measurement of \( e \). Flooding in tubes with non-smooth ends is represented in the analysis with higher values of the entrainment parameter than for tubes with smooth ends. At low liquid volumetric fluxes the theory overpredicts the experiment in Figure 5 for non-smooth tube ends. The disagreement can be attributed to the breakdown of liquid film flow or to the flow pattern which is not annular. More viscous liquids would tend to minimize this breakdown and the air-oil data in Figure 6 attest to this fact. At low liquid volumetric fluxes it is also possible to achieve a better comparison between the theory and the experiment by accounting for the variation of \( \phi \) with \( a \) in the analysis. This correction is, however, of the second order except at very high pressures.

![Figure 6 Comparison of air-oil experimental data with theory](image)

![Figure 7 The effect of liquid viscosity, system pressure and tube diameter on the distribution of adiabatic flooding fluxes](image)

For large values of the entrainment parameter, the homogeneous flow of liquid droplets and gas will not occur. At low gas volumetric fluxes it is reasonable to set this limit to be at \( e = .1 \).

Comparison of the experimental data of Tien et al. (6) for air-oil flooding with theory is illustrated in Figure 6. The data envelope is well represented by an entrainment parameter which is equal to .05. Lower values for \( e \) are expected for viscous fluids.

The results from adiabatic analysis are summarized in Figure 7 for the entrainment parameter equal to .1. As shown in the figure, the effect of increasing tube
diameter and liquid viscosity is to lower the gas volumetric flux for a given value of the liquid flux. With the increase in the system pressure, the flooding volumetric flux of the gas increases. This trend is reasonable, since with an increase of pressure the buoyancy force decreases and larger gas velocities are required to balance the forces at the inception of flooding.

The liquid droplets entrained in the gas are capable of introducing hysteresis into the flooding curves. This effect will be discussed in more detail in the next section.

Heat Transfer Flooding Solution

The nature of the heat transfer flooding results is illustrated in Figure 8. These results were generated by fixing the values of $N_t$, $D^*$, $N_l$, $Pr$ and $Bo$, and varying the subcooling number $S_{14}$ and the entrainment parameter $e$.

![Fig. 8 Condensation heat transfer flooding solution](image)

Fig. 8 Condensation heat transfer flooding solution

With condensation heat transfer, the liquid film volumetric flux is limited by equation (8b). The upper limit corresponds to infinitely long tube ($z/D=\infty$) where the liquid bulk temperature reaches the saturation temperature. At this point the flow becomes adiabatic and the adiabatic flooding solution should apply. The results in Figure 8 are in agreement with this conclusion. Furthermore, flooding with heat transfer occurs at the bottom of the tube where the liquid and vapor volumetric fluxes have largest values. With tubes of finite length, the effect of condensation is to raise the flooding curve above the one in adiabatic flow, and this effect is more pronounced at higher subcooling numbers. The condensing heat and mass flux has the effect of bringing stability to flooding in a similar manner that suction brings stability to the boundary layers. On the contrary, the entrained liquid droplets destabilize the flooding curve as in the adiabatic flow situation. The liquid droplets in the vapor core preserve the shape of the heat transfer flooding curve and bring more closely together this curve and the adiabatic flooding curve. This behavior is reasonable, since the liquid droplets do not contribute in latent heat towards the condensing heat transfer rate.

![Fig. 9 Effect of the interphase momentum transfer on the distribution of flooding fluxes](image)

Fig. 9 Effect of the interphase momentum transfer on the distribution of flooding fluxes

![Fig. 10 Heat transfer flooding solution for liquid subcooling of .14 and comparison with the experimental data of steam and water](image)

Fig. 10 Heat transfer flooding solution for liquid subcooling of .14 and comparison with the experimental data of steam and water

Figure 8 was generated by neglecting the effect of mass transfer on the interfacial shear stress in equation (15), and by setting the ratio of liquid-vapor interface velocity to the average liquid film velocity at 2. In Figure 9 these effects are investigated through the constants $I_f$ and $I_w$. As this figure shows, the effect of changing the velocity ratio ($I_w$) by a factor of 2 does not change results significantly. The effect of condensing mass flux on the interfacial shear stress (parameter $I_f$) is, however, very important. Its negative value implies that in counter-current condensing flows the liquid-vapor interface friction is lower than in
adiabatic flow. The experimental data presented below attest to this fact.

Heat transfer flooding curves for fixed $D^*$, $N$, $\delta_1$, $P$, $I$, $S$, and $v$ are illustrated in Figure 10 for different values of the tube length parameter $z/D$. These results were generated by connecting points of equal values of $z/D$ when $J_{11}$ is varied and with all other parameters remaining fixed. The presentation of results in the form of Figure 10 is important since it clearly shows the effect of tube geometry, and allows ready comparison of theory with the experimental data. For a given tube length and diameter, flooding states are represented on the line of constant $z/D$.

The experimental data of Liu and Collier (3), and Liu (14) for the counter-current condensation of steam on water are shown in Figures 10 to 12 for three different values of the liquid volumetric fluxes $J_{11}$. The liquid subcooling in Figure 10 is .14 and in Figure 11 it is .08. The data points were obtained by selecting $J_{11}$ and gradually increasing the steam flow from zero until the liquid was by-passed out of the tube and then by decreasing the steam flow back to zero again. For the tube length to diameter ratio of 10 in the experiment, flooding in Figure 10 should occur on the line $z/D=0.10$. This, however, occurs only for the lowest value of $J_{11}=1.05$. For higher values of $J_{11}$, the theory is in qualitative agreement with the experimental data and shows very little difference from the adiabatic flooding solution ($z/D=\infty$). For the low subcooling number in Figure 10, the theoretical solution for $z/D=10$ falls very closely to the line $z/D=\infty$ and is, therefore, not illustrated. At higher $J_{11}$, the theory underestimates the experiment while at low $J_{11}$, the theory can be made to agree with the experiment with the non-zero value of the entrainment parameter.

![Fig. 10 Comparison of small tube diameter experimental data with theory](image)

The effects of Prandtl number and tube length to diameter ratio on the flooding volumetric fluxes are illustrated in Figure 13 for the high subcooling number. Although the heat transfer correlation utilized in the model is outside the range of validity for this subcooling number, the qualitative solution behavior in Figure 13 should nevertheless be correct. The results illustrate that at higher subcoolings the Prandtl number can significantly affect the heat transfer flooding curve at high liquid volumetric fluxes. Lower Prandtl number fluids stabilize the flooding curve and can introduce hysteresis effect. The hysteresis effect in Figure 13 appears when the liquid volumetric flux is held fixed and the controlling variable is the vapor flux $J_{11}$. Thus starting with $J_{11}$ at point A and tube geometry $z/D=1$, and increasing $J_{11}$ to the point B on the curve with $P_{11}=3$, no change of the liquid flow occurs. At point B, however,
The effect of Prandtl number and tube length on the distribution of flooding fluxes is a slight change in $J_0$ shifts the operating point to the state at point C with considerably lower value of $J_1$. From point C to point D and from D to C, continuous changes in volumetric fluxes $J_0$ and $J_1$ are again possible. Upon decreasing $-J_0$ from D to A, the path followed is along DEFA, since continuous variation of $J_0$ can proceed from D to E. Very little hysteresis effect is, however, possible for the flooding curve with $z/D=1$ and $Pr_1=10$ on the path A'B'C'D'E'F'A'.

The hysteresis effect in flooding curves with heat transfer at high subcooling numbers also appears from equation (1) as demonstrated by Tien (2). The analysis presented herein uncovers very much richer physics than can be deduced from the flooding correlation in equation (1). For very long tubes ($z/D=0$) no hysteresis is possible for any Prandtl number or subcooling number, and for high subcooling numbers hysteresis is possible with finite tube lengths and low Prandtl numbers. No experimental evidence exists to justify the foregoing conclusions at high subcooling numbers. At low subcooling numbers the hysteresis effect can be explained by the liquid droplets entrainment as it has been discussed above.

The generally lower quantitative agreement in heat transfer results with the experimental data can be attributed to the poor representation of the local heat transfer process in the analysis. The heat transfer correlation for the average Nusselt number in equation (9) represents the average for the liquid subcoolings from $S_{11}=0.07$ to $S_{11}=14$ and for $J_1=0.05$ to $J_1=J_1$. This average is reflected in the local Nusselt number equation (2la) and, therefore, also in the analytic results presented above. Equation (9) is accurate within ±70% whereas the analytic results in Figures 10 and 11 reproduce the experimental data within ±30%. Since the experimental data for the small tube diameter in Figure 12 were not utilized in producing the correlation equation (9), the difference between the experiment and theory in this figure is 40%. The data at very low liquid volumetric fluxes in Figure 12 show considerable deviation from the theoretical prediction which is not the case for larger tube diameters in Figures 10 and 11. In adiabatic flow (Figures 5 and 6), the experimental data for low $D^*$ and for low $J_1$ are in good agreement with the theory. That such is not the case for the heat transfer data, can be attributed to the design peculiarities of the experiment, the methods utilized to introduce water and steam into the tube, etc. The heat transfer flooding results in Figures 10 to 13 should be viewed as the average representation of all steam-water data for which the correlation equation (9) is valid.

The average Nusselt number in equation (9) was deduced from the low subcooling number approximation equation (6') and the experimentally measured liquid bulk temperatures. For subcooling numbers greater than 1 such a representation is not adequate for long tubes, since the average Nusselt number can be considerably underpredicted as can be seen from Figure 2. Slight inaccuracy in the power representation of $z/D$ in equation (9) will also significantly change the numerical value of the bracket on the right of equation (2la) due to its large power dependance.

Future experimental work should attempt a more detailed study of the local heat transfer process of flooding in a tube. The analysis presented above is capable of incorporating any new developments of the local structure of the heat transfer process. The experimental data in Figures 10 to 12 were obtained by injecting water into the tube through a porous injector. Other methods of water injection might produce different results and need to be investigated. It would also be of value to extend the range of experimental data to higher subcoolings, wider range of liquid Prandtl numbers and to higher system pressures.

IV. SUMMARY AND CONCLUSIONS

An analysis has been developed to study flooding hydrodynamics and heat transfer in a tube. The hydrodynamic analysis considered local balance of gravity and viscous forces, and incorporated the effect of the liquid film-vapor core interface momentum transfer. The effect of liquid droplets entrained in the flow was taken into consideration through a simple model of the entrainment parameter.

The adiabatic flooding solution is found to depend on four parameters. These are the Bond number, $B^*$; ratio of gravity to viscous forces parameter, $N_0$; Sieder-Tate number or density ratio parameter, $Bo$; and the entrainment parameter $e$. Comparison of the theory with the experimental data of air-water and air-oil showed that best results are achieved for smooth tube ends with an entrainment parameter between 0.05 and 0.1. For sharp tube ends, good comparison is achieved at higher values of the liquid film volumetric flux. The effect of arbitrary parameters on flooding is illustrated by the adiabatic flooding curves in Figure 7. This parametric trend was found to be in agreement with physical observations.

In the presence of heat transfer, additional parameters are found to describe flooding in a tube. These are the subcooling number, $S_{11}$; liquid Prandtl number, $Pr_l$; and the tube length parameter, $z/D$. Confinement heat transfer introduces stability into the flooding curve in the sense that it raises its flooding volumetric fluxes. This stability is more pronounced for higher subcooling numbers, for lower Prandtl numbers and for lower $z/D$. The theory showed good qualitative and, on the average, good quantitative comparison with the experimental data of steam and water.

For steam-water flooding in a tube, the theory presented in this paper can be utilized, at the present,
as follows:

1. For low subcooling numbers, $S_r < 1.0$, the adiabatic flooding solution of Figure 7 applies with a non-zero value of the entrainment parameter.
2. For higher subcooling numbers, $S_r > 1.0$, it is conservative to utilize the adiabatic flooding solution with an entrainment parameter equal to zero. Equations (24) and (25) are very simple to use in this case.

Extrapolation of the theoretical results to high subcooling numbers revealed a hysteresis effect. This effect is more pronounced at higher subcooling numbers at lower Prandtl numbers and for shorter tubes.

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VI. REFERENCES