Volume One

Two-Phase Flow and Boiling Phenomena
Heat Transfer in Enclosures
Augmentation Heat Transfer
High Temperature Heat Exchangers
Compact Heat Exchangers
Air-Cooled Heat Exchangers
Gas Side Fouling in Heat Exchangers

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FOREWORD

The scope of thermal engineering ranges from basic principles to hardware application concerning the flow of heat and matter. It encompasses a broad fraction in the engineering discipline, including typically heat transfer, fluids engineering, combustion, vehicular propulsion, nuclear power generation and alternate energy sources. Since its infancy, research in thermal engineering has been mostly centered in the geographical area surrounding the Atlantic Ocean. In recent years, however, such research in the Pacific region has been increasing at a rapid pace, with Japan and the United States (who share both the Atlantic and Pacific Oceans) taking the lead. In order to stimulate interest and progress in research in thermal engineering in the Pacific area, the American Society of Mechanical Engineers and the Japanese Society of Mechanical Engineers have jointly sponsored the 1983 ASME-JSME Thermal Engineering Joint Conference held on March 20-24 in Honolulu. All countries and regions in the Pacific area are invited to participate but researchers from other geographical districts are equally welcome.

The Conference zeroes in on four important subjects: (i) the fundamentals of heat transfer with the stress on new theories, concepts and measuring techniques, (ii) heat exchangers with applications in energy systems, (iii) combustion and combustors for energy systems and (iv) thermal engineering problems relevant to energy systems. ASME has contributed fifteen sessions (including an open forum for oral presentations) with 125 articles covering item (ii) and part of items (i) and (iv). JSME has also contributed fifteen sessions with 110 articles covering item (iii) and part of items (i) and (iv). In addition, nine keynote lecturers are invited, five from ASME and four from JSME. All invited and general papers are presented in four volumes of the Conference proceedings. The articles contributed by ASME appear in Volumes 1 and 2, while those from JSME compose Volumes 3 and 4. All papers were reviewed in accordance with the standards of both sponsoring societies and thus were assured of high quality. In the case of Volumes 1 and 2, the ASME session chairmen should be credited for editing the respective chapters containing the papers that are presented in their sessions. The JSME Conference Organizing Committee has edited Volumes 3 and 4.

Volume 1 begins with a chapter on two-phase flow and boiling phenomena, a popular topic in heat transfer research. Interest in this subject never diminishes but continues to increase, as evidenced by the large number of articles presented. The chapter covers a broad spectrum including propagation of pressure waves and flow transition in two-phase flow, transient boiling, cavitation, dryout, droplet behavior, flashing, thermosyphone, nucleate and flow boiling, critical heat flux, burnout, impinging jets, transient behavior and correlations. Chapter 2 concerns a timely subject on natural convection in enclosures. The enclosures of various geometries are treated, including those filled with porous media. Experiments and numerical approaches represent two major tools in these studies. Various methods of heat transfer enhancement are applied in the third chapter, including the use of treated surfaces, fins, ribs, rough surfaces, turbulence promoters, swirl flow devices and surface vibration. The last four chapters are concerned with heat exchangers for high temperature applications, of compact-and-light weight type, of air-cooled type, and the problems of air-side fouling. Applications of compact type to boiling, dehumidification/cooling systems and systems with heat generation are unique. Of air-cooled type, effort is directed toward augmentation, correlations, overall heat transfer coefficient, effect of flow maldistribution, and dry/wet operation.

Volume 2 contains seven chapters and five keynote lectures. The first six chapters are concerned with the contemporary applications of heat transfer, such as fluidized/packed systems, underground conversion, geothermal utilization, underground media, ocean thermal energy conversion (OTEC) and alternate energy sources. The last chapter is composed of late papers covering miscellaneous topics on two phase flow, thermal stability, packed beds and heat exchangers. The keynote lectures were delivered by well-known young experts on the topics of separated forced convection, particle transport properties and dispersion in turbulent flow, heat and mass transfer in combustion, shell-side condensation, and boiling in porous media, respectively.

Volume 3 consists of five chapters and two keynote lectures covering some aspects of heat transfer fundamental and applications to power generating systems. Grossly diversified subjects in natural, forced and combined convection feature the uniqueness of the volume. The chapter on conduction is characterized by various applications, while the condensation chapter treats film condensation, augmentation, noncondensable gas effect, direct-contact condensation and reflux condensation in thermosyphons. The articles on advanced thermal power generation systems treat the performance of two-phase flow turbines and the regenerator effectiveness of a Stirling engine. The chapter on heat transfer in nuclear reactors focuses on two major concerns, namely the performance and safety of nuclear reactors. It is fortunate that two internationally well-known scholars on critical heat flux and boiling heat transfer, respectively, are willing to share their authoritative knowledge and life-long experience on the subjects in their keynote lectures.

The articles on combustion, combustors for energy systems and thermal engineering problems relevant to high-temperature energy systems are presented in Volume 4. There are nine chapters dealing with combustion in general, turbulent combustion, furnace combustion, combustion in Diesel engines, combustion for fuel economy, alternative fuel engines, gas turbine components and applications, thermal radiation and heat transfer in general. The volume is capped by two keynote lectures on combustion and hydrogen fuel engines. The combustion-related chapters cover a broad spectrum in combustion ranging from the basic principles to application as well as fuel economy.

The editors wish to express their gratitude to all those who contributed to the publication of the proceedings. They include the authors, the reviewers of the papers, the ASME session organizers, the JSME Conference Committee members, the session chairmen and vice chairmen, the ASME and JSME Headquarters personnel and finally, Professor H. Echigo, Ms. Ling H. Yang and Ms. Mimi L. I. Yang for their assistance. The Conference has provided the opportunity for the exchange of ideas and experiences among the researchers in various areas of thermal engineering. We hope that the proceedings will prove to be a valuable reference for future research.

Wen-Jei Yang, Ann Arbor
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CONTENTS

VOLUME ONE

TWO-PHASE FLOW AND BOILING PHENOMENA

AN ACCELERATION WAVE MODEL FOR THE SPEED OF PROPAGATION OF SHOCK WAVES IN A BUBBLY TWO-PHASE FLOW

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ABSTRACT

The propagation speeds of shock waves in bubbly two-phase fluid mixtures have been investigated by means of an acceleration wave model. By assuming that the two-phase fluid mixture is nonconducting and that the acceleration wave can propagate as a shock wave, an analytical expression is derived for the speed of propagation of this wave which depends on the speeds of sound of both phases, on the densities of phases, on the volume fraction of the dispersed phase, and on the model for the virtual mass coefficient in the constitutive equation for the interphase momentum transfer. It is shown that the model for the virtual mass coefficient significantly affects the speed of propagation of the acceleration wave and that the best comparison between the analysis and the experimental data for the speeds of propagation of shock waves is achieved when the coefficient depends on the volumetric fraction of the dispersed phase.

1. INTRODUCTION

A small amplitude compressive disturbance in a two-phase mixture of liquid and gas propagates at the speed of sound and the propagation speed of this sound wave can be smaller than the speed of sound in either phase [1]. Furthermore, the speed of sound in two-phase flows is significantly affected by the flow regime (such as stratified, slug and bubbly flow) and by the pressure of the mixture [1]. As the amplitude of the compressive disturbance in a bubbly mixture is increased, the speed of sound behind the disturbance becomes greater than in the front of it and this disturbance or wave front readily steepens and forms a shock wave [2]. The strength of the shock wave depends on the volumetric fraction of the gas phase in the mixture and it is larger at lower gas volumetric fractions where the rate of sonic velocity change with the gas volumetric fraction is greatest.

The propagation speed of a finite amplitude pressure disturbance does not only depend on the strength of the disturbance and flow regime, but also on whether this disturbance is generated into an initially stationary or into a moving two-phase mixture. In the latter case, the prevailing distribution of the pressure gradient in the flow field can be of considerable importance. The experimental studies of shock waves which are propagating into stationary air-water and steam-water bubbly mixtures close to the atmospheric pressure were undertaken by Campbell and Pitcher [3], Wijngaarden [4], Miyazaki, Fujii-E and Suita [5], Mori, Hijikata and Komine [6], and by Noordzij and Wijngaarden [7]; whereas the propagation speeds of shock waves into moving air-water bubbly mixtures close to the atmospheric pressure were investigated by Padmanabhan and Martin [2], and by Akagawa et al. [8]. From these experiments the following conclusions can be drawn about the structure of shock waves in bubbly two-phase flows:

i) At very small gas volumetric fractions, the shape of the shock wave changes along the duct [7]. The wavefront starts off with a steep pressure profile between the front and the back of the wave, and the back of the wave is characterized by a high frequency pressure oscillation (type A shocks in Ref. 7). Further downstream along the duct, the wave front is characterized by a gradual pressure change and no pressure oscillations are observed behind the wave (type C shocks). Noordzij and Wijngaarden [7] attribute this change of the wavefront pressure profile along the duct to the relative motion of bubbles and liquid.

ii) In a long tube in which a mixture of air and water is flowing, Padmanabhan and Martin [2] observed no type A shocks when the valve at the downstream end of the tube was quickly closed. However, they did observe that compressive disturbances of moderate strengths transform into steep shock waves and that the structure of shock waves did not change along the tube. The absence of type A shock waves is attributed to large valve-closing times, and the absence of change in the wave structure along the tube is attributed to the lower viscosity of water in the experiments of Ref. 2 than of water/glycerine in the experiments of Ref. 7.

iii) Akagawa et al. [8] also report type A and C shocks in the immediate vicinity of the valve at the downstream end of the tube and classify these shocks according to the gas volumetric fraction α (type A shocks correspond to $\alpha < .075$ and type C shocks correspond to $\alpha > .13$). It is important to note that the shock classification in Ref. 8 is based on the observations in the vicinity of the downstream valve, while in Ref. 7 it is based on the observations along the tube.

The speed of propagation of shock waves in bubbly two-phase mixtures is greater than the acoustic speed and it is attributed to the strength of the shock P_1/P_o, where P_1 and P_o are pressures behind and in front of the shock wave respectively. Based on the onedimensional and homogeneous flow model with $\rho_g/\rho_\ell << 1$, and assuming that the gas phase behaves as an isothermal ideal gas, the following speed of propagation w of shock waves can be obtained ([3], [7], [8]):

$$w = a_o\sqrt{\frac{P_1}{P_o}} \quad , \quad a_o^2 = \frac{P_o}{\alpha(1-\alpha)\rho_\alpha} \quad , \tag{1}$$

where a_o is the speed of sound in an isothermal and homogeneous two-phase flow. Eq. (1) compares favorably with the available experimental data of air-water and steam-water bubbly flows at low pressures and for low values of gas volumetric fractions ([3], [5], [8]). At high pressures, the assumption that the gas behaves as an ideal gas is expected to break down, and at higher gas volumetric fractions α, the assumptions that the flow is homogeneous and flow pattern bubbly are wrong. In these circumstances, then, Eq. (1) is not expected to apply.

From the above, it is clear that although a great deal of progress has been made in the investigation of shock waves in two-phase flows, there still remains a number of issues to be cleared up. In particular, the problem dealing with the growth and decay of finite amplitude pressure disturbances and the relation between the speed of propagation of these disturbances with various flow regimes are poorly understood. The purpose of this paper is to propose an expression for the propagation speed of shock waves in bubbly two-phase flows which is based on an acceleration wave model. It is shown that such a model is indeed reasonable and, therefore, that the acceleration wave model can form a basis for a subsequent investigation of the growth and decay of finite amplitude disturbances in bubbly two-phase flows.

2. ANALYSIS

2.1 Governing Equations

An acceleration wave model for the speed of propagation of finite amplitude disturbances is constructed by assuming that the acceleration wave is a singular surface across which the velocity and temperature remain continuous but the acceleration suffers a discontinuity. For the purpose of analyzing the speed of propagation of such waves, I will utilize the conservation laws, jump conditions across the singular surfaces, and constitutive equations as developed by Dobran [9,10].

The two-phase fluid model in Ref. 9 is constructed by volume averaging of the instantaneous field equations for each phase, and the constitutive equations in Ref. 10 account for the temperature gradient in the mixture, for velocity gradients, for the viscous drag, and for the virtual mass effects. To each point of space $\mathbf{x}$ corresponds a deformation function $\chi_{\beta\kappa}$ of phase β, $\beta=1,2$, such that

$$\mathbf{x} = \chi_{\beta\kappa}(\mathbf{X}_\beta, t) \quad , \tag{2}$$

where $\mathbf{X}_\beta$ is the place of a particle of phase β in the reference configurations κ_β. The velocity and acceleration of phase β are

$$\mathbf{v}_\beta \equiv \frac{\partial\chi_{\beta\kappa}(\mathbf{X}_\beta,t)}{\partial t} \quad , \quad \grave{\mathbf{v}}_\beta \equiv \frac{\partial^2\chi_{\beta\kappa}(\mathbf{X}_\beta,t)}{\partial t^2} \quad . \tag{3}$$

The partial density of phase β is $\bar{\rho}_\beta = \alpha_\beta\rho_\beta$, where α_β is the volumetric fraction and ρ_β is the mass density of phase β. The density and velocity of the two-phase mixture are then defined by the following equations:

$$\rho = \sum_{\beta=1}^{2} \bar{\rho}_\beta = \sum_{\beta=1}^{2} \alpha_\beta\rho_\beta \quad , \quad \rho\mathbf{v} = \sum_{\beta=1}^{2} \bar{\rho}_\beta \mathbf{v}_\beta \quad , \tag{4}$$

and the diffusion velocity $\mathbf{u}_\beta$ is defined as follows:

$$\mathbf{u}_\beta = \mathbf{v}_\beta - \mathbf{v} \ . \tag{5}$$

A moving surface in two-phase flow is represented by

$$f(\mathbf{x}, t) = 0$$

with $\mathbf{x}$ lying on this surface. The unit normal vector $\mathbf{n}$ on the moving surface and the velocity $\mathbf{W}$ of this surface are represented by the following equations:

$$\mathbf{n} = \frac{\nabla f}{(\nabla f\cdot\nabla f)^{\frac{1}{2}}} \quad , \quad \mathbf{W} = -\frac{\partial f}{\partial t}\frac{\mathbf{n}}{(\nabla f\cdot\nabla f)^{\frac{1}{2}}} \quad . \tag{6}$$

If Ω is a function of $(\mathbf{x}, t)$ and it is continuous except on a discontinuous surface, then the jump of Ω is defined by

$$[\Omega] \equiv \Omega^+ - \Omega^- \quad , \tag{7}$$

where Ω^+ is the limit of Ω at a fixed time t as the singular surface is approached from the side towards which $\mathbf{n}$ is directed and Ω^- is the corresponding limit of Ω as the singular surface is approached from the other side. The conservation and balance equations for phase β, $\beta=1,2$, in the absence of phase change are from Ref. 9:

Mass: $$\grave{\bar{\rho}}_\beta + \bar{\rho}_\beta\nabla\cdot\mathbf{v}_\beta = 0 \tag{8}$$

$$[\bar{\rho}_\beta(\mathbf{v}_\beta - \mathbf{w})]\cdot\mathbf{n} = 0 \tag{9}$$

Linear Momentum: $$\bar{\rho}_\beta\grave{\mathbf{v}}_\beta = \nabla\cdot\bar{\mathbf{T}}_\beta + \bar{\rho}_\beta\mathbf{b}_\beta + \hat{\mathbf{p}}_\beta \tag{10}$$

$$[\rho_\beta\mathbf{v}_\beta((\mathbf{v}_\beta-\mathbf{w})\cdot\mathbf{n}) - \bar{\mathbf{T}}_\beta\mathbf{n}] = \mathbf{o} \tag{11}$$

Angular Momentum: $$\hat{\mathbf{M}}_\beta = \bar{\mathbf{T}}_\beta - \bar{\mathbf{T}}_\beta^T \tag{12}$$

Energy: $$\bar{\rho}_\beta\grave{\varepsilon}_\beta = tr(\bar{\mathbf{T}}_\beta^T\nabla\mathbf{v}_\beta) - \nabla\cdot\bar{\mathbf{q}}_\beta + \bar{\rho}_\beta r_\beta + \hat{\varepsilon}_\beta \tag{13}$$

$$[\bar{\rho}_\beta(\varepsilon_\beta + \tfrac{1}{2}\mathbf{v}_\beta\cdot\mathbf{v}_\beta)(\mathbf{v}_\beta-\mathbf{w}) + \bar{\mathbf{q}}_\beta - \bar{\mathbf{T}}_\beta^T\mathbf{v}_\beta]\cdot\mathbf{n} = 0 \tag{14}$$

Entropy: $$\bar{\rho}_\beta\grave{s}_\beta + \nabla\cdot\left(\frac{\bar{\mathbf{q}}_\beta}{\bar{\Theta}_\beta}\right) - \frac{r_\beta\bar{\rho}_\beta}{\bar{\Theta}_\beta} + \hat{s}_\beta \geq 0 \tag{15}$$

$$[\bar{\rho}_\beta s_\beta(\mathbf{v}_\beta - \mathbf{w})\cdot\mathbf{n} + \frac{1}{\bar{\Theta}_\beta}\bar{\mathbf{q}}_\beta\cdot\mathbf{n}] \geq 0 \quad , \tag{16}$$

where $\overline{\mathbf{T}}_\beta$ is the stress tensor, $\mathbf{b}_\beta$ is the body force vector, $\hat{\mathbf{p}}_\beta$ is the momentum supply or interaction term, ε_β is the internal energy, $\overline{\mathbf{q}}_\beta$ is the heat flux vector, r_β is the heat generation rate per unit volume of the mixture, $\hat{\varepsilon}_\beta$ is the energy supply or interaction term, s_β is the entropy, Θ_β is the temperature, and $\hat{s}_\beta$ is the entropy supply. In Eq. (11), $\overline{\mathbf{T}}_\beta\mathbf{n}$ denotes the product of a linear transformation $\overline{\mathbf{T}}_\beta$ (a second order tensor) and a vector and is, thus, a vector, and in Eq. (14), $\mathrm{tr}(\overline{\mathbf{T}}_\beta{}^T\nabla\mathbf{v}_\beta)$ denotes the trace operation on the linear transformation formed by the product of linear transformations $\overline{\mathbf{T}}_\beta{}^T$ and $\nabla\mathbf{v}_\beta$. The subscript T denotes the transpose, and the backward prime affixed to a quantity of phase β denotes the material derivative of that phase. For a quantity ψ_β, for example, we have:

$$\grave{\psi}_\beta \equiv \frac{\partial\psi_\beta}{\partial t} + \nabla\psi_\beta \cdot \mathbf{v}_\beta \quad . \tag{17}$$

The constitutive equations which will be utilized in this article to study the propagation speeds of acceleration waves assume: 1) no phase change, 2) both fluids are at a single but nonuniform temperature, 3) viscous effects in the constitutive equation for $\overline{\mathbf{T}}_\beta$ are absent, 4) relative velocity and virtual mass effects are not negligible, and 5) the two-phase mixture is in a state close to the equilibrium, i.e. the linearized constitutive equations are applicable. In this case we have from Ref. 10:

$$\hat{\mathbf{p}}_\beta = -\gamma_\beta \nabla\Theta - \xi_{\beta 1}(\mathbf{v}_1-\mathbf{v}_2) - \Delta_{\beta 1}(\grave{\mathbf{v}}_1-\grave{\mathbf{v}}_2) \tag{18}$$

$$\hat{\mathbf{M}}_\beta = \mathbf{O} \tag{19}$$

$$\overline{\mathbf{T}}_\beta = -\overline{\pi}_\beta \mathbf{I} \tag{20}$$

$$\overline{\mathbf{q}}_\beta = -\kappa_\beta\nabla\Theta - \zeta_{\beta 1}(\mathbf{v}_1-\mathbf{v}_2) - \nu_{\beta 1}(\grave{\mathbf{v}}_1-\grave{\mathbf{v}}_2) - \overline{\rho}_\beta s_\beta \Theta \mathbf{u}_\beta \tag{21}$$

$$\hat{\varepsilon}_\beta = 0, \quad \hat{s}_\beta = 0 \tag{22}$$

$$\varepsilon_\beta = \varepsilon_\beta(\Theta,\overline{\rho}_\beta), \quad s_\beta = s_\beta(\Theta,\overline{\rho}_\beta), \quad \overline{\pi}_\beta = \overline{\pi}_\beta(\Theta,\overline{\rho}_\beta) \tag{23}$$

$$\psi_\beta = \varepsilon_\beta - \Theta s_\beta \ , \ d\psi_\beta = -s_\beta \, d\Theta + \frac{\overline{\pi}_\beta}{\overline{\rho}_\beta^2} d\overline{\rho}_\beta \quad , \tag{24}$$

where $\mathbf{I}$ is the unit linear transformation, $\overline{\pi}_\beta$ is the partial pressure, and the coefficients γ_β, $\xi_{\beta 1}$, $\Delta_{\beta 1}$, κ_β, $\zeta_{\beta 1}$, and $\nu_{\beta 1}$ depend on the equilibrium state properties of two-phase flow (Θ, $\overline{\rho}_1$ and $\overline{\rho}_2$). In order for Eqs. (18) - (23) to satisfy the second law of thermodynamics Eq. (15) we must have (see Ref. 10 for details):

$$\kappa_\beta \geq 0 \ ; \quad \beta = 1,2 \tag{25}$$

$$\xi_{11} \geq 0, \quad \xi_{11} + \xi_{21} = 0 \tag{26}$$

$$\zeta_{11} = -\Theta(\gamma_1 + \overline{\rho}_1 s_1)\frac{\overline{\rho}_2}{\rho} \tag{27}$$

$$\zeta_{21} = \Theta(\gamma_2 + \overline{\rho}_2 s_2)\frac{\overline{\rho}_1}{\rho} \tag{28}$$

$$\Delta_{11} \geq 0 \ , \ \Delta_{11} + \Delta_{21} = 0, \ \gamma_1 + \gamma_2 = 0 \quad . \tag{29}$$

The conditions specified by Eqs. $(26)_2$, $(29)_2$ and $(29)_3$ are only valid for dispersed two-phase flows where interfacial and nonlocal effects can be neglected [9, 10,11].

2.2 The Acceleration Wave

An acceleration wave is a singular surface across which

$$[\Theta] = 0 \ , \ [\mathbf{v}_\beta] = \mathbf{o} \ , \quad \mathbf{a}_\beta \equiv [\grave{\mathbf{v}}_\beta] \neq \mathbf{o} \ , \tag{30}$$

where $\mathbf{a}_\beta$ is the amplitude vector of the acceleration wave. For a quantity $\psi(\mathbf{x},t)$ which is continuous across the discontinuity, $[\psi] = 0$, and a theorem due to Maxwell (Ref. 12, Sec. 180) states that

$$\left[\frac{\partial\psi(\mathbf{x},t)}{\partial t}\right] = -[\nabla\psi(\mathbf{x},t)] \cdot \mathbf{n}\, w \tag{31}$$

$$[\nabla\psi(\mathbf{x},t)] = -\left[\frac{\partial\psi(\mathbf{x},t)}{\partial t}\right]\frac{\mathbf{n}}{w} \quad , \tag{32}$$

where w is the normal speed of the singular surface, $\mathbf{w} = \mathbf{n}w$.

Since the velocity $\mathbf{v}_\beta$ is continuous across the acceleration wave (Eq. $(30)_2$), Maxwell's theorem Eq. (32) yields:

$$[\nabla\mathbf{v}_\beta] = -\mathbf{a}_\beta \otimes \frac{\mathbf{n}}{w} \tag{33}$$

$$[\nabla\cdot\mathbf{v}_\beta] = -\mathbf{a}_\beta \cdot \frac{\mathbf{n}}{w} \quad , \tag{34}$$

where $\otimes$ denotes the tensor product of the vectors $\mathbf{a}_\beta$ and $\mathbf{n}$. From Eqs. (9), (11) and (20) the densities and pressures are continuous across the acceleration wave, i.e.

$$[\overline{\rho}_\beta] = 0, \quad [\overline{\mathbf{T}}_\beta] = -[\overline{\pi}_\beta]\mathbf{I} = \mathbf{O} \ . \tag{35}$$

Using Eqs. $(35)_{1,2}$, (19), $(30)_2$, (12) and $(22)_1$ in Eq. (14), we obtain that the heat flux is continuous across the wave

$$[\overline{\mathbf{q}}_\beta]\cdot\mathbf{n} = 0 \ . \tag{36}$$

Forming the jump of Eq. (8) and utilizing Eqs. $(35)_1$ and (34) the results are

$$[\grave{\overline{\rho}}_\beta] = -[\overline{\rho}_\beta\nabla\cdot\mathbf{v}_\beta] = \overline{\rho}_\beta \frac{\mathbf{a}_\beta\cdot\mathbf{n}}{w} \tag{37}$$

and substituting $\psi = \overline{\rho}_\beta$ in Eq. (32) and using (37) the following result is obtained:

$$[\nabla\overline{\rho}_\beta] = -[\grave{\overline{\rho}}_\beta]\frac{\mathbf{n}}{w} = -\overline{\rho}_\beta \frac{\mathbf{a}_\beta\cdot\mathbf{n}}{w^2}\mathbf{n} \quad . \tag{38}$$

Using Eqs. (24), (19), (20), (8) and $(23)_2$ in the energy Eq. (13) results in the following simple expression for the energy equation:

$$\overline{\rho}_\beta C_{V\beta}\grave{\Theta} - \frac{\Theta}{\overline{\rho}_\beta}\left(\frac{\partial\overline{\pi}_\beta}{\partial\Theta}\right)_{\overline{\rho}_\beta}\grave{\overline{\rho}}_\beta = -\nabla\cdot\overline{\mathbf{q}}_\beta + \overline{\rho}_\beta r_\beta \ , \tag{39}$$

where the specific heat at constant volume $C_{V\beta}$ is defined by

$$C_{V\beta} \equiv \Theta\left(\frac{\partial s_\beta}{\partial\Theta}\right)_{\overline{\rho}_\beta} = \left(\frac{\partial\varepsilon_\beta}{\partial\Theta}\right)_{\overline{\rho}_\beta} \quad , \tag{40}$$

and

$$\grave{\Theta} \equiv \frac{\partial\Theta}{\partial t} + \nabla\Theta\cdot\mathbf{v}_\beta \quad . \tag{41}$$

In order to keep the model as simple as possible, I will also assume that the two-phase mixture is nonconducting and that no heat generation in the mixture exists. Thus

$$\bar{\mathbf{q}}_\beta \equiv \mathbf{o} \ , \qquad r_\beta \equiv 0 \quad . \tag{42}$$

Utilizing these assumptions in Eq. (39), forming the jump of the same equation, and using in the resulting expression Eq. (31) in order to eliminate $[\grave{\Theta}]$ and $[\grave{\bar\rho}_\beta]$, we obtain that the jump in the temperature gradient is related to the jump in the density gradient by

$$[\nabla\Theta] = \frac{\Theta}{\bar\rho_\beta^2\, c_{V\beta}}\left(\frac{\partial\bar\pi_\beta}{\partial\Theta}\right)_{\bar\rho_\beta}[\nabla\bar\rho_\beta] \ . \tag{43}$$

From Eqs. $(23)_3$ and (43), and from the fact that the density and the partial derivatives of the pressure with respect to the temperature and density are continuous across the wave, it is easy to show that

$$[\nabla\bar\pi_\beta] = \left(\frac{\partial\bar\pi_\beta}{\partial\bar\rho_\beta}\right)_{s_\beta}[\nabla\bar\rho_\beta] \quad , \tag{44}$$

where use was made of the following thermodynamic identity:

$$\left(\frac{\partial\bar\pi_\beta}{\partial\bar\rho_\beta}\right)_{s_\beta} = \left(\frac{\partial\bar\pi_\beta}{\partial\bar\rho_\beta}\right)_{\Theta} + \frac{\Theta}{\bar\rho_\beta^2\, c_{V\beta}}\left(\frac{\partial\bar\pi_\beta}{\partial\Theta}\right)^2_{\bar\rho_\beta} \ . \tag{45}$$

Forming the jump of the momentum supply Eq. (18) and using Eqs. (30), $(35)_1$, and (43), we obtain

$$[\hat{\mathbf{p}}_\beta] = -\gamma_\beta\frac{\Theta}{\bar\rho_\beta^2\, c_{V\beta}}\left(\frac{\partial\bar\pi_\beta}{\partial\Theta}\right)_{\bar\rho_\beta}[\nabla\bar\rho_\beta] - \Delta_{\beta1}(\mathbf{a}_1-\mathbf{a}_2) \ . \tag{46}$$

The assumption that the two-phase mixture is adiabatic (Eq. $(42)_1$) requires that in Eq. (21)

$$\kappa_\beta=0, \quad \nu_{\beta1}=0, \quad \zeta_{\beta1}(\mathbf{v}_1-\mathbf{v}_2)+\bar\rho_\beta\, s_\beta\,\Theta\,\mathbf{u}_\beta = \mathbf{o} \ , \tag{47}$$

and since $\mathbf{v}_1 - \mathbf{v}_2 = \mathbf{u}_1 - \mathbf{u}_2$, Eq. $(47)_3$ is reduced to

$$\begin{pmatrix} \zeta_{11}+\bar\rho_1 s_1\Theta & -\zeta_{11} \\ \zeta_{21} & \bar\rho_2 s_2\Theta-\zeta_{21}\end{pmatrix}\begin{pmatrix}\mathbf{u}_1\\ \mathbf{u}_2\end{pmatrix} = \begin{pmatrix}\mathbf{o}\\ \mathbf{o}\end{pmatrix} \tag{48}$$

which for the nontrivial solution requires that the determinant of the square matrix vanishes. Carrying out this computation and using the constitutive Eqs. (27), (28) and $(29)_{2,3}$ it follows that

$$\gamma_1 = \gamma_2 = 0 \quad . \tag{49}$$

Finally, forming the jump of the momentum Eq. (10) and utilizing Eqs. (20), $(35)_1$, $(30)_3$, (46) and (38), we obtain

$$\bar\rho_\beta\,\mathbf{a}_\beta = \left(\frac{\partial\bar\pi_\beta}{\partial\bar\rho_\beta}\right)_{s_\beta}\frac{\bar\rho_\beta}{w^2}(\mathbf{a}_\beta\cdot\mathbf{n})\mathbf{n} - \Delta_{\beta1}(\mathbf{a}_1-\mathbf{a}_2) \ . \tag{50}$$

By decomposing the acceleration amplitude vector $\mathbf{a}_\beta$ into tangential and normal components to the singular surface, Eq. (50) can be investigated for the speeds of propagation of both the transverse and longitudinal waves. A substitution of $\mathbf{a}_\beta = (\mathbf{a}_\beta\cdot\mathbf{t})\mathbf{t}$ into Eq. (50), where $\mathbf{t}$ is the unit tangent vector to the singular surface, yields:

$$\begin{pmatrix}\bar\rho_1+\Delta_{11} & -\Delta_{11}\\ \Delta_{21} & \bar\rho_2-\Delta_{21}\end{pmatrix}\begin{pmatrix}(\mathbf{a}_1\cdot\mathbf{t})\mathbf{t}\\ (\mathbf{a}_2\cdot\mathbf{t})\mathbf{t}\end{pmatrix} = \begin{pmatrix}\mathbf{o}\\ \mathbf{o}\end{pmatrix} \ . \tag{51}$$

Setting the determinant of the square matrix equal to zero and utilizing $\Delta_{21} = -\Delta_{11}$ (Eq. $(29)_2$) results in the expression:

$$\Delta_{11} = -\frac{\bar\rho_1\bar\rho_2}{\bar\rho_1+\bar\rho_2} < 0$$

which is inconsistent with Eq. $(29)_1$. It, therefore, follows from Eq. (51) that $\mathbf{a}_1\cdot\mathbf{t} = \mathbf{a}_2\cdot\mathbf{t} = 0$, and: no transverse acceleration waves can exist in a dispersed fluid mixture of two phases.

Substituting $\mathbf{a}_\beta = (\mathbf{a}_\beta\cdot\mathbf{n})\mathbf{n}$ into Eq. (50), the propagation equations for the longitudinal waves in two-phase flow become:

$$\begin{pmatrix}\bar\rho_1+\Delta_{11}-\frac{\bar\rho_1}{w^2}\left(\frac{\partial\bar\pi_1}{\partial\bar\rho_1}\right)_{s_1} & -\Delta_{11}\\ \Delta_{21} & \bar\rho_2-\Delta_{21}-\frac{\bar\rho_2}{w^2}\left(\frac{\partial\bar\pi_2}{\partial\bar\rho_2}\right)_{s_2}\end{pmatrix}\begin{pmatrix}(\mathbf{a}_1\cdot\mathbf{n})\mathbf{n}\\ (\mathbf{a}_2\cdot\mathbf{n})\mathbf{n}\end{pmatrix} = \begin{pmatrix}\mathbf{o}\\ \mathbf{o}\end{pmatrix} \tag{52}$$

Again, utilizing Eq. $(29)_2$ and setting in the above equation the determinant of the square matrix equal to zero, the speed of propagation of longitudinal acceleration waves becomes:

$$w^2 = (A_1 \pm \sqrt{A_2})/A_3 \quad , \tag{53}$$

where

$$A_1 = \bar\rho_1 c_1^2(\bar\rho_2+\Delta_{11}) + \bar\rho_2\, c_2^2(\bar\rho_1+\Delta_{11})$$

$$A_2 = [\bar\rho_1 c_1^2(\bar\rho_2+\Delta_{11}) - \bar\rho_2 c_2^2(\bar\rho_1+\Delta_{11})\,]^2 + 4\bar\rho_1\bar\rho_2 c_1^2 c_2^2\Delta_{11}^2$$

$$A_3 = 2[\bar\rho_1\bar\rho_2+\Delta_{11}(\bar\rho_1+\bar\rho_2)] \quad ,$$

and

$$c_1^2 \equiv \left(\frac{\partial\bar\pi_1}{\partial\bar\rho_1}\right)_{s_1} \quad , \qquad c_2^2 \equiv \left(\frac{\partial\bar\pi_2}{\partial\bar\rho_2}\right)_{s_2} \quad . \tag{54}$$

The discriminant in Eq. (53) can be also written in the following equivalent way:

$$A_2=[\bar{\rho}_1C_1^2(\bar{\rho}_2+\Delta_{11})+\bar{\rho}_2C_2^2(\bar{\rho}_1+\Delta_{11})]^2 - 4\bar{\rho}_1\bar{\rho}_2C_1^2C_2^2[\bar{\rho}_1\bar{\rho}_2+\Delta_{11}(\bar{\rho}_1+\bar{\rho}_2)] \qquad (55)$$

which shows that in Eq. (53) there are two physical solutions for the speed of propagation of acceleration waves. When $\Delta_{11}\to 0$ these two speeds approach the speeds C_1 and C_2 given by Eq. (54).

3. DISCUSSION

The speed of propagation of an acceleration wave in a two-phase flow is expressed by Eq. (53). It is first observed in this equation that in the limit of single phase flow, the speed of an acceleration wave is reduced to the speed of sound in that phase. Second, when $\Delta_{11}\to 0$, the two-phase mixture becomes in a sense ideal and then the acceleration waves propagate at the speeds C_1 and C_2 given by Eq. (54). For a two-phase bubbly flow, however, $\Delta_{11} \neq 0$ as a number of studies confirm (see Refs. 13 and 14), and Eq. (53) shows that the propagation speed of an acceleration wave can be significantly lower than the speed of sound in either phase (see Figs. 1 and 2). For a small pressure disturbance in a two-phase flow, the acceleration wave speed is equivalent to the speed of sound of the two-phase mixture since no experimental evidence exists that small pressure disturbances grow into larger ones to form, for example, shock waves. This indicates that a two-phase mixture effectively damps small pressure disturbances and that this damping comes about primarily because of the relative motion between the phases. The large amplitude pressure disturbance, however, has the effect of producing a shock wave as discussed above, and the shock wave propagates at a greater speed than the speed of sound in the mixture. Since no assumption was made in the derivation of Eq. (53) for the size of the acceleration disturbance, this equation should, therefore, be able to predict the speed of propagation of shock waves.

To investigate the possibility for Eq. (53) to model the speeds of propagation of shock waves, constitutive equations must be supplied in this equation for C_1, C_2 and Δ_{11}. A very reasonable assumption is that $C_1 = a_1$ and $C_2 = a_2$, where a_1 and a_2 are the speeds of sound in phases 1 and 2 respectively, i.e.

$$a_1^2=\left(\frac{\partial P_1}{\partial \rho_1}\right)_{s_1}, \quad a_2^2=\left(\frac{\partial P_2}{\partial \rho_2}\right)_{s_2}, \qquad (56)$$

where P_1 and P_2, and ρ_1 and ρ_2, are pressures and densities of the two phases. The above assumption is reasonable in view of the fact that

$$\bar{\pi}_\beta=\alpha_\beta P_\beta, \quad \bar{\rho}_\beta=\alpha_\beta\rho_\beta, \quad \alpha_1+\alpha_2=1 \qquad (57)$$

$$\left(\frac{\partial\bar{\pi}_\beta}{\partial\bar{\rho}_\beta}\right)_{s_\beta}=\left(\frac{\partial P_\beta}{\partial \rho_\beta}\right)_{s_\beta}+\frac{P_\beta}{\rho_\beta+\alpha_\beta/\left(\frac{\partial\alpha_\beta}{\partial\rho_\beta}\right)_{P_\beta}}\simeq\left(\frac{\partial P_\beta}{\partial\rho_\beta}\right)_{s_\beta}, \qquad (58)$$

since, in many situations of practical interest,

$$\left(\frac{\partial\alpha_\beta}{\partial\rho_\beta}\right)_{P_\beta}\simeq 0.$$

In a bubbly two-phase fluid mixture, the coefficient $\Delta_{\beta 1}$ in Eq. (18) is referred to as the virtual mass coefficient. It depends on the equilibrium state properties of two-phase flow (Θ, $\bar{\rho}_1$, and $\bar{\rho}_2$), and in the literature ([13], [14]) it is given by

$$\Delta_{11} = \alpha_2\rho_1 C_{VM}, \qquad (59)$$

where α_2 is the volumetric fraction of the dispersed phase and C_{VM} is regarded as a function of α_2 [14]. Upon substituting in Eq. (53)

$$\Delta_{11}=\alpha\rho_\ell C_{VM}, \quad \bar{\rho}_1=\alpha\rho_g, \quad \bar{\rho}_2=(1-\alpha)\rho_\ell$$

$$C_1 = a_g, \quad C_2 = a_\ell,$$

where α is the volumetric fraction of gas bubbles, we obtain

$$\left(\frac{w}{a_g}\right)^2_{1,2} = (B_1 \pm \sqrt{B_2})/B_3, \qquad (60)$$

where

$$B_1=\frac{\rho_g}{\rho_\ell}\left(1+\frac{\alpha}{1-\alpha}C_{VM}\right)+\frac{a_\ell^2}{a_g^2}\left(\frac{\rho_g}{\rho_\ell}+C_{VM}\right)$$

$$B_2=\left[\frac{\rho_g}{\rho_\ell}\left(1+\frac{\alpha}{1-\alpha}C_{VM}\right)-\frac{a_\ell^2}{a_g^2}\left(\frac{\rho_g}{\rho_\ell}+C_{VM}\right)\right]^2+4\frac{\alpha}{1-\alpha}\frac{\rho_g}{\rho_\ell}\frac{a_\ell^2}{a_g^2}C_{VM}^2$$

$$B_3=2\left[\frac{\rho_g}{\rho_\ell}+C_{VM}\left(\frac{\alpha}{1-\alpha}\frac{\rho_g}{\rho_\ell}+1\right)\right].$$

The two physical solutions for the wave speeds in Eq. (60) are denoted by w_1 and w_2. For an air-water and steam-water mixture close to the atmospheric pressure where $\rho_g \ll \rho_\ell$, Eq. (60) predicts that w_1 is independent of α and C_{VM}, and that $w_1 \simeq a_\ell$. The second wave speed w_2 in the same equation is, however, not only much less than either a_ℓ or a_g, but it also strongly depends on C_{VM} as is illustrated in Fig. 1 with the dashed lines, and this dependence is more pronounced at lower values of C_{VM}. For a single spherical bubble moving through an infinite liquid the theory predicts that $C_{VM} = .5$, but for a bubbly two-phase mixture C_{VM} is regarded to be less than .5 and a function of α [14].

The dependence of C_{VM} on α can be ascertained from the experimental data of the speed of propagation of shock waves in two-phase flows as is shown in Fig. 1. The data in this figure pertain to the air-water/glycerine two-phase mixtures in which shock wave fronts propagate at an atmospheric pressure into stationary two-phase mixtures. At a small value of α, the speed of propagation of shock wave is very sensitive on its value, whereas at a larger value of α, this speed levels off to a constant. A very reasonable accord between the experimental data in Fig. 1 and w_2 can be achieved by the following simple representation of C_{VM} in Eq. (60):

$$C_{VM} = .3\tanh(4\alpha). \qquad (61)$$

Utilizing this value of C_{VM}, further comparison of the theory with the experimental data is shown in Fig. 2 for two cases: 1) for the case when the shock wave propagates into a flowing air-water mixture, and 2) for the case when the shock wave propagates into a stationary steam-water mixture. The prediction of the experimental

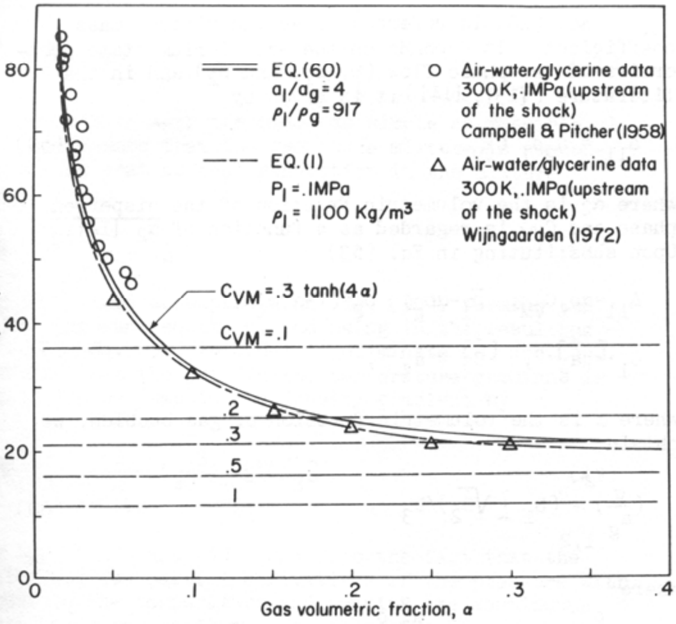

FIG. 1 COMPARISON OF THE ANALYTICAL AND EXPERIMENTAL SHOCK SPEEDS

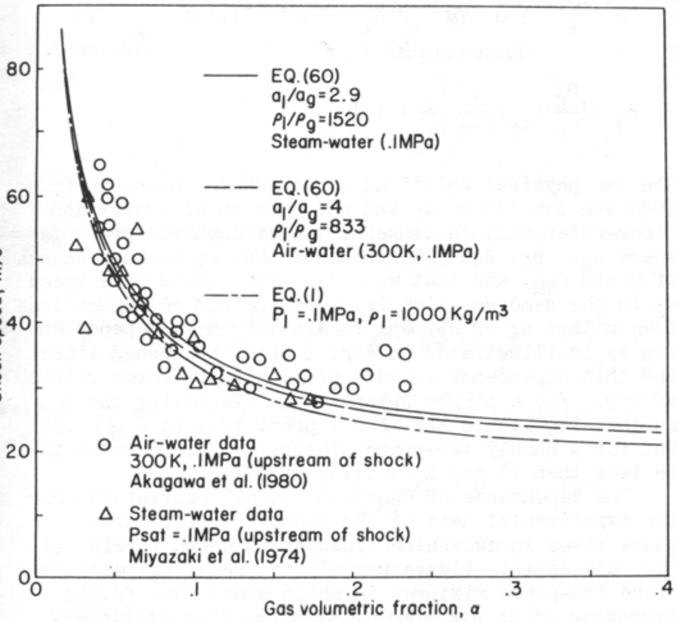

FIG. 2 COMPARISON OF THE ANALYTICAL AND EXPERIMENTAL SHOCK SPEEDS

data in Fig. 2 is very reasonable except possibly of the data of Akagawa et al. [8] at large values of α. However, since the experimental data in Ref. 8 were scaled to the atmospheric pressure, this scaling might not be accurate at large α. Without such a scaling each data point would correspond to a different pressure of the wave front.

Also shown in Figs. 1 and 2 are the predictions of shock wave speeds from Eq. (1). In both figures this prediction is very reasonable and has the advantage over Eq. (60) of being simpler. At higher pressures, where the assumptions that $\rho_g << \rho_\ell$ and that the gas obeys the ideal gas low become incorrect, Eq. (1) ceases to be valid.

4. CONCLUSIONS

The acceleration wave model presented in the paper is able to predict the speed of propagation of shock waves in bubbly two-phase flows reasonably well provided that an account is taken in the theory for the dependence of the coefficient C_{VM} in the virtual mass coefficient $\Delta_{\beta 1}$ on the volumetric fraction of the dispersed phase. Having established this, the acceleration wave model discussed in the paper can be utilized to study the growth and decay of finite amplitude disturbances in two-phase bubbly flows. Through such a study, it should be possible to prove the existence of a critical disturbance where any initial disturbance with an amplitude greater than the critical one always grows into a shock wave, while a disturbance with an initial amplitude less than the critical always decays. The existence of a critical amplitude should be consistent with the relative motion and dissipative effects in two-phase flows.

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