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PROSPECTS FOR THE GLOBAL VOLCANIC SIMULATION OF VESUVIUS

ABSTRACT. — Accurate forecast of future volcanic events at Vesuvius requires detailed understanding of the past behavior of the volcano and extrapolation of this behavior into the future. To accomplish this goal, it is suggested to develop a Global Volcanic Simulator for Vesuvius through interdisciplinary scientific efforts from volcanology, geology, geophysics, petrology, mathematical and physical modeling, and computer science. Volcanological, geological, petrological, and geophysical studies are required to understand the origin and composition of deposits, magma and lava flows, presence of aquifers, strength, elasticity and plasticity of magmas, lavas, surrounding rocks and soils, etc. Research in applied mathematics requires the development of accurate models at the microscale and macroscale levels for the magma, lava, soil, and pyroclastic dispersions, and the development of constitutive equations for viscoelastic and viscoplastic materials with several phases, inhomogeneities, and anisotropic behavior. Research in applied mathematics also requires research in fluid mechanics, solid mechanics, and numerical methods for solving large number of governing equations in an efficient manner and accounting for steep variations in the field variables. A significant research effort is also required in computer science to take advantage of available and future computers in order to solve efficiently a large system of non-linear, strongly-coupled equations which govern the behavior of magma, lava, soil, pyroclastic dispersion, etc. The development of a Global Volcanic Simulator for Vesuvius does not only depend on the scientific breakthroughs in several fields of science, which will be inevitably achieved in the near future, but also on bringing together the responsible agencies for protecting the Vesuvian area populations and physical scientists and naturalists, whose views on the scientific method are often different, to work toward a common goal. Moreover, the development of a volcanic simulator for Vesuvius will also serve to educate young researchers in several disciplines such as those mentioned above. The reasons for developing a Global Volcanic Simulator for Vesuvius are so strong that any opposition leading to this development is destined to fail, since such failures very often occurred in the past when opposing scientific progress.

KEY WORDS - Vesuvius, volcanic eruption, computer simulation.

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1. INTRODUCTION

In 1990, the National Volcanic Group of Italy (GNV) submitted to the Ministro della Protezione Civile a report entitled «Eruptive Scenario of Vesuvius». This report synthesized the available scientific information on Vesuvius and underlined the enormous potential for the volcanic risk associated with the entire Vesuvian area. In 1992, a commission formed by the Minister produced guidelines [1] for the evaluation of the volcanic risk associated with the Vesuvian area, establishing that there are about 700,000 persons who may be exposed to the risk. Early in 1993 GNV produced a call for proposals to promote a first three-year plan of research on Vesuvius with the objective to obtain, through interdisciplinary approach, a quantification of the hazard of the volcano.

Since 1990, the italian volcanological community has been summoned to pursue the quantification of volcanic hazard in the Vesuvian area by developing a Global Volcanic Simulator for Vesuvius [2-4]. The realization of such a goal requires the incorporation of volcanological, petrological, geophysical, and other data in producing physical models of different volcanic processes at Vesuvius which will be subsequently combined into a simulator for implementation on a computer (Fig. 1*a*). This computer program can then be used to produce different eruption scenarios or hazard-zonation maps for the Vesuvian area.

Although all of the above reports call for an urgent quantification of volcanic hazard at Vesuvius, no significant step has so far been taken by the responsible Italian agencies to promote the development of a volcanic simulator for Vesuvius. By mid 1993, GNV decided on a three-year research program for Vesuvius but did not regard the physical modeling approach as a tool for the evaluation of volcanic hazard at Vesuvius. This is unfortunate, since a Global Volcanic Simulator for Vesuvius could serve to both forecast volcanic hazard and prevent substantial damage to the Vesuvian area populations. It is also unfortunate that current volcanic hazard forecasting methods based on data collected from past eruptions and present volcanic activity and extrapolations may be doomed to be completely incorrect as it happened, for example, with the Lagrange's view mechanics and chaos, the use of Newtonian mechanics in quantum mechanics, etc.

The objectives of this paper are: (1) to summarize the research which is needed to develop a global volcanic simulator for Vesuvius in order to use this simulator to establish hazard-zonation maps for the Vesuvian area, and (2) to indicate the difficulties associated with the development of the simulator. A much more detailed description of a Global Volcanic Simulator can be found elsewhere [4] and only a short summary will be presented in the paper. After a brief description of the Vesuvian volcanic complex and inferred functioning of

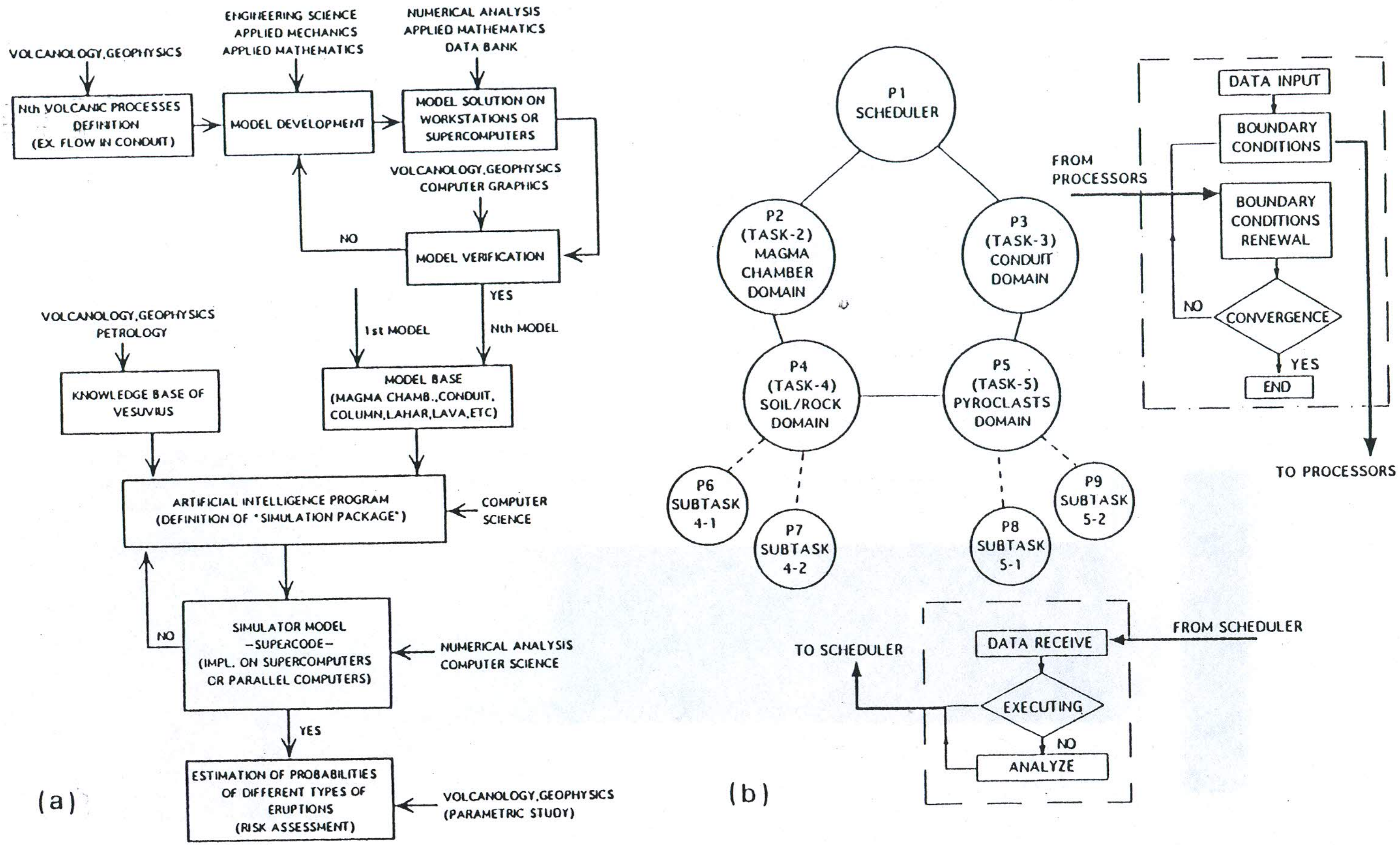


Fig. 1. – (*a*) Global Volcanic Simulator development stages. (*b*) A possible processor assignment to various tasks in the Global Volcanic Simulator.

the volcano, the required interdisciplinary research efforts leading to the development of the simulator will be discussed. It will be concluded that the development of a Global Volcanic Simulator for Vesuvius would not only benefit the Vesuvian area population but also the scientific community because it requires a significant progress in many branches of science, including volcanology, petrology, and geology. Any opposition leading to the development of the simulator must fail, as this is an opposition to both the education of young researchers and scientific progress which has very often failed throughout the history of science.

2. THE VESUVIAN VOLCANIC SYSTEM

2.1 *Morphology and past eruptions of Vesuvius.*

The convergence of the Eurasian and African plates is responsible for the volcanic activity in the central and southern parts of Italy. The Vesuvian region includes the Somma-Vesuvius volcanic complex and is located between the opening Tyrrhenian basin to the west and the westward-migrating Apennine compressive front to the east [5]. In the Campanian region, the volcanic activity apparently started about 600,000 years ago at Roccamonfina [6], 300,000 years ago at Somma, and 50,000 years ago at Phlegraean Fields which is responsible for the emplacement of the Campanian Ignimbrite about 35,000 years ago [7]. Monte Somma's activity terminated about 17,000 years ago with the birth of Vesuvius within the Somma caldera. Vesuvius is a cone-shaped stratovolcano with a summit crater about 450 m in diameter and 330 m deep [5].

For the past 35,000 years, the Somma-Vesuvius complex has exhibited various types of activities. Large-scale plinian eruptions Codola, Sarno, Basal, Greenish, Lagno Amendolare, Mercato, Avellino, and Pompei each erupted several cubic kilometers of material and occurred every few centuries to millenia, whereas the intermediate-scale subplinian eruptions in 412 and 1631 A.D. occurred every few centuries each erupting about 0.1 km^3 of material [8]. The smaller-scale strombolian and effusive events occurred every few decades, and it appears that these events normally follow the plinian and subplinian eruptions until the conduit closes [4]. A common feature of the plinian eruptions is that they were intermittently interrupted due to partial column collapses producing pyroclastic surges and flows, and terminated with the interaction of magma with water from underground aquifers [9-12]. The plinian and subplinian eruptions from Avellino about 3400 years ago to 1631 are all characterized by the emission of highly differentiated trachytic and phonolitic magmas [13, 14]. The pumice-fall deposits of Avellino, Pompei, and 1631 eruptions consists of white phonolite at the base and gray tephritic phonolite at the top [15, 16]. The deposits of plinian eruptions contain limestones, various thermometamorphosed marble and skarn lithic ejecta which suggest the location of the magma chamber and/or magma fragmentation levels within the Mesozoic carbonate basement between 3 and 5 km below the volcano [17]. The subplinian deposits do not contain carbonate lithic ejecta, suggesting that the magma reservoirs and/or magma fragmentation levels were located above the Mesozoic carbonate basement. From the petrological and volcanological evidence it appears that the volcanic activity at Vesuvius can be associated with a periodic magma supply to the Vesuvian feeding system [14]. At present this result is, however, poorly constrained by thermodynamics, geophysics, and thermofluid-dynamics [4].

2.2 *Definition of reference eruptions.*

Test cases or reference eruptions of Vesuvius are required for initiating computer simulations and verifying the Global Volcanic Simulator [4]. The initial conditions of the simulator consist of the thermofluid-dynamic properties of magma, gas, and pyroclasts, and structural mechanics properties of the volcanic edifice. These conditions, when supplemented by the boundary conditions at the domain boundaries for all subsequent times, determine the evolution of the volcanic complex in time and space. If a simulation is initiated from the closed-conduit state of volcano, then this simulation should be able to predict the subsequent volcanic eruptions. It is important to note that the effects of the initial conditions on the evolution of the volcanic system tend to be forgotten in time, but that this is not true for the boundary conditions which in effect determine the system evolution in time and space.

Based on the foregoing considerations, the proper initial conditions of the Vesuvian volcanic complex required for simulations should be obtained when the conduit closed after the A.D. 79 Pompei eruption in order to avoid the difficulty of specifying the volcanic system conditions during the open conduit condition [4]. This event apparently occurred around 205 A.D. and requires further studies [18]. The 1631 subplinian eruption of Vesuvius is a well-known event in the Vesuvian area and forms a good test case for verifying the predictions of the simulator. This eruption produced a plinian column, caldera collapse, pyroclastic flows and surges, and generated lahars and mud flows [16]. In order for the simulator to predict this eruption starting from the initial conditions, the simulator would have to simulate magma supply, differentiation, and crystallization in the magma chamber which lead to conduit opening and subsequent closure after the driving pressure for magma discharge is reduced and the eruption is terminated. The 1631 eruption did not, however, completely close the conduit of Vesuvius, and the simulator would have to predict this very important event which probably occurred in 1944.

3. Required interdisciplinary research

3.1 *Volcanological, geological, geophysical, and thermodynamic parametrization of magmas.*

The development of a global volcanic simulator requires volcanological, geological and geophysical data and parametrization of magmas to define the volcanic system, verify the computer simulations, and forecast future volcanic events at Vesuvius. These data pertain to the definition of initial and boundary conditions of the volcanic complex and to the identification of the 1631 eruption parameters [4].

The identification of initial and boundary conditions requires the establishment of substructural conditions of the volcano, such as magma supply, magma differentiation, and volcanic edifice conditions. Magma supply conditions pertain to the establishment of possible mineralogy of the source giving rise to the primary Vesuvian magmas, determination of thermodynamic phase diagrams of rocks in the magma production zone of Vesuvius and establishment of geothermal gradient(s) in this region in the presence or absence of H_2O and CO_2, establishment of thermofluid-dynamic constraints of magma transport in the supply region, magma assimilation, and geochemical and isotopic characterization of magma. Magma differentiation conditions pertain to the knowledge of the location, geometry, size, and composition of the melt and solids in the magma chamber following the conduit closure after A.D. 79. This requires volcanological studies of tephra deposits, and determination of kinetics parameters of all crystallizing phases and rheological properties of magma in the presence of crystals and vapor phases. The volcanic edifice conditions involve the properties which define the thermal and mechanical states of rocks surrounding the conduit(s) and magma chamber. In particular, the establishment of a geological model of Vesuvius is necessary for a detailed specification of the volcanic edifice.

The identification of the 1631 eruption parameters required for simulator verification calls for a topographic reconstruction of the Vesuvius prior to the eruption in 1631, detailed studies of stratigraphic layers and their spatial and temporal correlations to establish the composition and granulometry of the erupted material and association of these data with the conditions at the vent during the course of the eruption, provenance and characterization of lithics, identification of the location and thermal states of aquifers, and reconstruction of the time-wise behavior of mass flow-rates during the plinian and pyroclastic flow phases of the eruption. Geophysical studies at Vesuvius are urgently needed and should involve the realization of high-resolution and three-dimensional seismic tomography aimed at identifying the structures below the central portion of the cone of Vesuvius and below the Somma caldera, and the establishment of gravity, magnetometry, and electrical data. In particular, a geophysical model of Vesuvius should also be developed.

The thermodynamic parametrization studies of magmas should have the objective of parametrizing the Vesuvian magmas in terms of composition, pressure, and temperature where the time-scale of molecular relaxation processes may be important. The parametrization studies of Vesuvian magmas should involve the spatial and temporal characteristics associated with macroscopic magmatic processes determined from thermofluid-dynamic modeling. These studies are essential for constraining magmatic processes and forecasting future volcanic events by the simulator.

3.2 *Physical, mathematical, and computer modeling.*

The forecast of volcanic events at Vesuvius requires global modeling of the volcanic system [4]. This modeling should adequately resolve the thermofluid-dynamic processes of magma mixing, differentiation, and crystallization in the magma chamber, changes in the magma chamber geometry with time due to the inflow and outflow of magma and changing stresses of surrounding rocks, magma ascent along the conduit(s) and interaction with conduit's walls, structural response of the volcanic edifice to magma chamber and conduit processes, and distribution of erupted products in the atmosphere and along the slopes of the volcano. The global model should therefore simulate all relevant physical processes below and above the surface of the Earth well and efficiently.

A volcanic system such as Vesuvius may be conveniently divided into different parts or domains characterized by unique properties or characteristic physical phenomena. These parts may consist of magma chamber, conduit, soil or country rock, and pyroclasts domains. The magma chamber domain consists of an open system for mass, momentum, and energy transfer where multiphase and multicomponent flow phenomena involving crystallization, exsolution, and melting occur. The conduit domain can be characterized by propagating fractures induced by magma, wherein the magma may exsolve the dissolved gases, fragment into pyroclasts, and interact with conduit walls and surrounding aquifers. The soil or rock domain encloses the magma chamber and conduit domains and is characterized by the elastic, plastic, and nonhomogeneous media whose evolution depends on the surrounding domains. The pyroclasts domain involves mixing of tephra with the atmosphere and interaction of pyroclastic products with the topography of Vesuvius.

Modeling of elastic, plastic, and nonhomogeneous media, and multicomponent and multiphase flow phenomena in magma chamber, conduit(s), and atmosphere requires the development of appropriate physical models and associated constitutive equations. Magma chamber modeling must account for several crystallizing phases, crystallization kinetics, and gas exsolution, whereas the conduit modeling must account for crack propagation due to intruding magma, magma-water interaction, magma fragmentation, and erosion of conduit walls. The rock or soil modeling should be based on the deformation maps which characterize the elastic, plastic, or viscoelastic characteristics of rocks at different pressures and temperatures. Furthermore, the closing phase of an eruption requires modeling of the conduit and magma chamber wall collapse which can be accomplished by using a multiphase modeling approach and deformation maps [4]. The pyroclasts domain modeling must involve modeling of pyroclasts of several granulometric particle classes, mixing of magmatic gases with the atmosphere and interaction with pyroclasts, particle-particle interactions and two-way turbulence coupling between the gas and particulate phases, condensation of water vapor in the atmosphere, and erosion of the slopes of Vesuvius.

The physical laws employed for modeling of volcanic transport processes of rocks, soil, magma, and gas consist of balance of mass, and first and second Euler's and Cauchy's laws of motion and thermodynamics. To these equations must also be added the appropriate constitutive and transport equations expressing the structural and turbulent characteristics of different materials and flows [4, 19, 20]. The solutions of these equations depend on the appropriate initial and boundary conditions established from volcanological, petrological, and geophysical data. The models must then be verified with these data before they can be employed for modeling of different types of volcanic processes. Once such a modeling capability is achieved, the models can then be combined together into a volcanic simulation package (Global Volcanic Simulator) and the global modeling predictions can be verified with test cases or reference eruptions (Fig. 1*a*). After this verification process, the simulator can be used to perform parametric studies to establish hazard-zonation maps of different volcanic events.

The global simulation of Vesuvius will depend on the effectiveness of combining different domain models into an overall computational scheme. Such a scheme is illustrated in Fig. 1*b* and involves a multiprocessor computer environment whereby to each processor of a multiprocessor computer or to each computer in a distributed computer environment is assigned a single domain or part of this domain. The interface between the domains is performed by a scheduler/supervisor processor which updates inter-domain boundary conditions and assigns computation times to free processors. The appropriate division of computational tasks among processors in a parallel computational environment will require careful optimization studies involving physical models, numerical algorithms, and computer architectures [4].

4. VOLCANIC HAZARD ASSESSMENT BY THE SIMULATOR

The ultimate utility of a Global Volcanic Simulator of Vesuvius is to produce volcanic hazard-zonation maps and educate the population of the impeding danger of the volcano. The hazard-zonation maps can be produced by performing simulations from the initial conditions after the conduit closure following the A.D. 79 eruption with different parameters of the volcanic complex which represent the mean values and variances of the volcanological, petrological, and geophysical data pertaining to the volcano. The uncertainties of the Vusuvian system parameters will thus produce uncertainties in volcanic event forecasting, with the minimization of parameter variances leading to a better forecast. It is important to stress that the volcanic hazard-zonation maps produced in this manner will be based on probabilities associated with different volcanic events (lava flow, pyroclastic flow, tephra fall, lahars, etc.) at different times in different regions in the Vesuvian area [4].

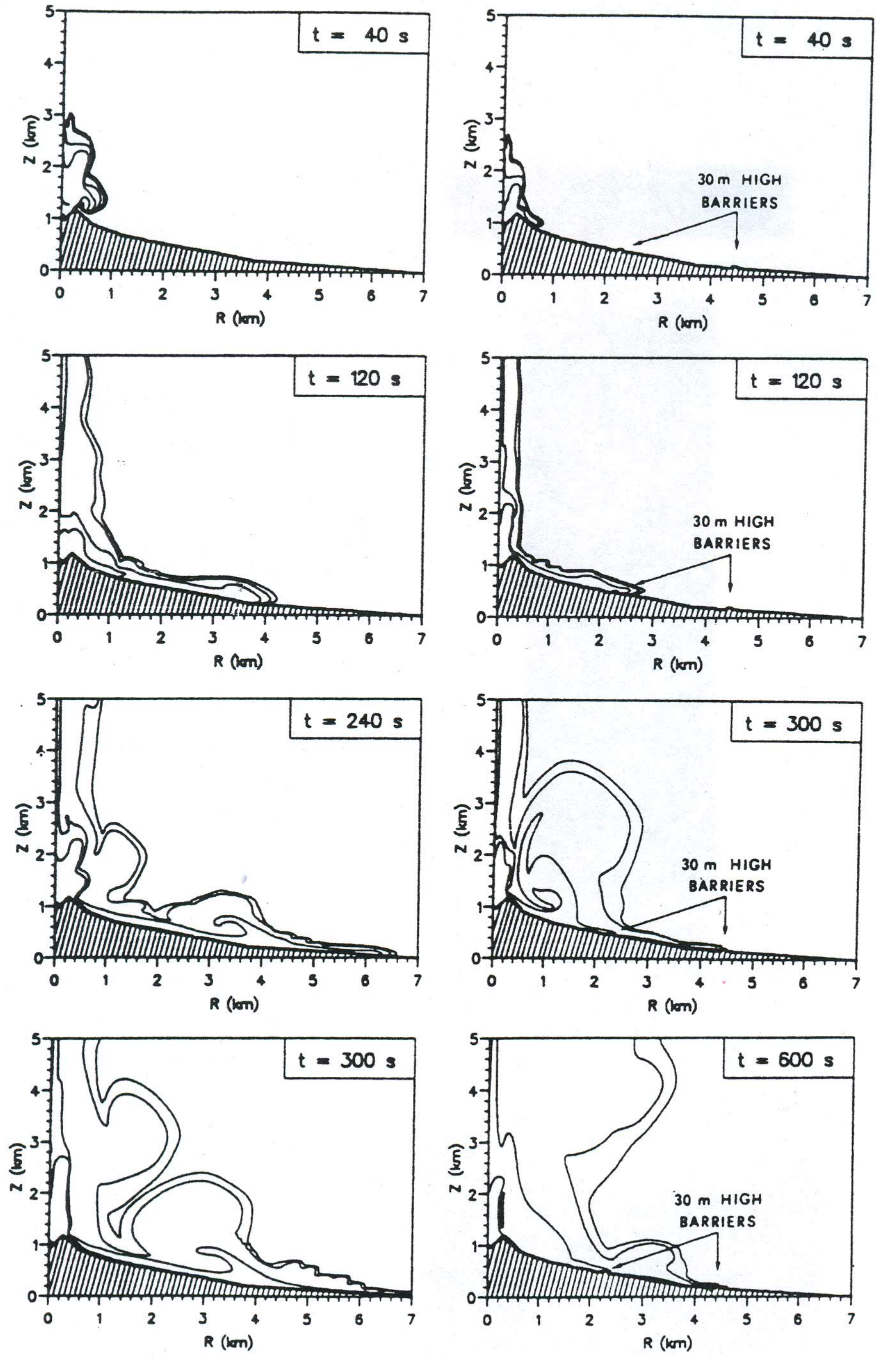
Perhaps the most useful purpose of a Global Volcanic Simulator of Vesuvius is to educate the population of the Vesuvian area of the impending danger of the sleeping giant. The simulator can be used by schools to educate the children, by policy makers, city planners, insurance companies, etc., to prepare for the future, construct the appropriate barriers on the slopes of the volcano, enforce new building codes to withstand the fury of certain volcanic events, and construct effective escape routes. Figure 2 illustrates the present modeling capabilities of volcanic columns [21] and shows the catastrophic effects which can be produced by pyroclastic flows in less than 300 s in the Vesuvian area following a volcanic column collapse. The collapse of a volcanic column such as that which destroyed Herculaneum and Pompei in A.D. 79 is capable to destroy everything in a 7 km radius from the volcano in several minutes (Fig. *2a*) and its destructive power cannot be diminished even by the Monte Somma relief [22]. An order of magnitude smaller volcanic event than that during the A.D. 79 eruption producing pyroclastic flows at Vesuvius may, however, be arrested for a sufficiently long time by building barriers at strategic locations on the slopes of Vesuvius. Figure *2b* illustrates that 30 m high barriers constructed at about 2.5 and 4.5 km from the vent can slow down and arrest a pyroclastic flow for at least 10 minutes which may be a sufficient time for saving many lifes.

5. PROSPECTS FOR GLOBAL SIMULATION OF VESUVIUS

The development of a Global Volcanic Simulator for Vesuvius depends on future scientific breakthroughs in several fields of science which will be inevitably achieved in the foreseeable future, as well as bringing together physical scientists and naturalists working toward a common goal whose views on the scientific method are often different. After several years of effort working in the Italian volcanological community, my experience is that the latter goal is much more difficult to achieve. The difficulties arise first of all from different backgrounds of researchers and from the mistrust which, after an initial enthusiasm, many researchers show for disciplines unknown to them. To this it can also be added the fact that the Italian volcanological community is not used to organize and carry out research such as the one required for the development of a volcanic simulator. There are many underdeveloped countries in the world where the volcanic risk is real and which can benefit from a volcanic simulator but cannot afford to produce it. Italy, on the other hand, has a similar volcanic hazard problem and can afford to build a simulator. Such a Global Volcanic Simulator for Vesuvius would also be a significant investment in science, since it would serve a two-fold purpose: a better hazard assessment in the Vesuvian area and the formation of researchers who must deal with this problem from an interdisciplinary point of view.

The present and future well being of the populations in the Vesuvian area depends on the maximum use of talented individuals from all branches of science. Unless the contributions of these individuals are recognized, encouraged, and supported, the interests and survival of Vesuvian area populations cannot

Fig. 2. – Computer simulation of volcanic plumes and pyroclastic flows moving along the southern slopes of Vesuvius. (*a*) Column collapse from a large-scale eruption typical of the A.D. 79 gray eruption phase of Vesuvius [22]. At 300 s from the beginning of column collapse a phoenix cloud is seen at about 3.5 km from the vent and the flow reaches the Tyrrhenian Sea 7 km away from the vent. (*b*) Column collapse from a medium-scale eruption of Vesuvius [22] and the effects of 30 m high barriers placed at about 2.5 and 4.5 km from the vent. At 600 s shown in the figure, the pyroclastic flow is arrested by the lower barrier. Contour levels shown in the figures represent pyroclasts volumetric fractions and, starting from the outer flow region, correspond to 10^{-8}, 10^{-6}, 10^{-4}, 10^{-2} and 10^{-1}.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0
1
2
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(a)

(b)

Fig. 2.

be insured. Both the Italian and international communities must do everything possible to avoid the consequences of a future catastrophe of the most hazardous volcano in Italy.

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